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THE SUBCRITICAL REACTOR WITH SOURCES

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ABSTRACT

The mathematical formulation of the subcritical reactor with extraneous sources is presented in detail. The solutions are presented for a two-group approximation including epithermal fission and the relationship to the usual critical reactor problem is discussed. Application of the equations to an arbitrarily complicated geometry is outlined, and an IBM-7090 program for the solution in a bare cylindrical reactor is described.

THE SUBCRITICAL REACTOR WITH SOURCES

I. Introduction

The calculation of neutron flux distributions in subcritical chain reacting systems containing extraneous sources is occasionally encountered in reactor analysis. The mathematical development of this problem is given below and the relationship to the more frequently encountered problem of a critical reactor is pointed out. The development is carried out in the two-group approximation commonly used in reactor analysis, including epithermal fission, and the equations for the fast neutron distributions are given. This distribution is not presented and epithermal fission is not considered in frequently consulted texts.[1-3]

II. General Formulation

We write the two-group diffusion equations for a time independent neutron distribution in the form:

$$D_1\nabla^2\phi_1(\underline{r}) - \Sigma_1\phi_1(\underline{r}) + \frac{k_2}{p}\Sigma_2\phi_2(\underline{r}) + k_1\Sigma_1\phi_1(\underline{r}) + S(\underline{r}) = 0, \quad (1)$$

$$D_2\nabla^2\phi_2(\underline{r}) - \Sigma_2\phi_2(\underline{r}) + p\Sigma_1\phi_1(\underline{r}) = 0. \quad (2)$$

Where it has been assumed that the reactor is homogeneous and uniform and the group constants have the following definitions:

D_1 = fast group diffusion coefficient,

D_2 = thermal group diffusion coefficient,

Σ_1 = fast group total cross section (absorption plus removal),

Σ_2 = thermal group absorption cross section,

p = resonance-escape probability, the number of neutrons thermalized per initial source neutron in an infinite reactor,

k_2 = number of neutrons produced, per initial source neutron, by thermal fissions in an infinite reactor,

k_1 = number of neutrons produced, per initial source neutron, by nonthermal fissions in an infinite reactor.

In order to solve Eqs. (1) and (2) we assume that the functions $\phi_1(\underline{r})$ and $\phi_2(\underline{r})$ may be expressed as a linear combination of the solutions of Laplace's equation

$$\nabla^2\psi_n(\underline{r}) + B_n^2\,\psi(\underline{r}) = 0, \tag{3}$$

where the functions $\psi_n(\underline{r})$ are defined in the same interval (volume) as ϕ_1 and ϕ_2 and satisfy the same symmetry conditions and boundary conditions at the outer boundary. This expansion will be valid if the functions ψ_n form a complete orthogonal set. The numbers B_n in Eq. (3) compose a discrete set of distinct eigenvalues of the operator ∇^2 for the specified boundary conditions and are the only values for which the functions ψ_n exist.

We write for ϕ_1 and ϕ_2

$$\phi_1(\underline{r}) = \sum_{n=1}^{\infty} A_n\psi_n(\underline{r}), \tag{4}$$

$$\phi_2(\underline{r}) = \sum_{n=1}^{\infty} C_n\psi_n(\underline{r}), \tag{5}$$

and for the source function

$$S(\underline{r}) = \sum_{n=1}^{\infty} S_n\psi_n(\underline{r}). \tag{6}$$

We now substitute Eqs. (3), (4), (5), and (6) into (1) and (2) and obtain

$$\sum_{n=1}^{\infty}\left\{[-D_1B_n^2 - \Sigma_1(1-k_1)]\,A_n + \frac{k_2}{p}\,\Sigma_2C_n + S_n\right\}\psi_n(\underline{r}) = 0, \tag{7}$$

$$\sum_{n=1}^{\infty} \left\{[-D_2B_n^2 - \Sigma_2]\, C_n + p\Sigma_1 A_n\right\} \psi_n(\underline{r}) = 0. \tag{8}$$

If the above equations are multiplied successively by $\psi_1(\underline{r})$, $\psi_2(\underline{r})$, etc., and integrated over the reactor volume, then the assumed orthogonality of the functions ψ_n over this interval results in our obtaining an equation for each value of n in which one of the integrals $\int \psi_n^2(\underline{r})\, d\underline{r}$ is a common factor in each. Upon cancellation of these factors we obtain the following equations for each value of n:

$$[-D_1B_n^2 - \Sigma_1(1-k_1)]\, A_n + \frac{k_2}{p}\, \Sigma_2 C_n + S_n = 0, \tag{9}$$

$$[-D_2B_n^2 - \Sigma_2]\, C_n + p\Sigma_1 A_n = 0. \tag{10}$$

III. Source Free Problem

At this point it is instructive to digress for a moment to examine the solutions of Eqs. (9) and (10) for the source free problem. In this case all of the S_n are zero and the above equations are pairs of linear homogeneous equations for the coefficients A_n and C_n. In order that a non-trivial solution exist it is necessary that the determinant of the coefficients vanish. That is,

$$\begin{vmatrix} -D_1B_n^2 - \Sigma_1(1-k_1) & \frac{k_2}{p}\, \Sigma_2 \\ p\Sigma_1 & -D_2B_n^2 - \Sigma_2 \end{vmatrix} = 0,$$

from which we obtain

$$\frac{k_2}{(1 + L^2B_n^2)(1 + \tau B_n^2)} + \frac{k_1}{(1 + \tau B_n^2)} = 1. \tag{11}$$

In Eq. (11) we have set

$$\frac{D_2}{\Sigma_2} = L^2 \qquad\qquad \frac{D_1}{\Sigma_1} = \tau.$$

Since all of the B_n are distinct it is clear that Eq. (11) can be satisfied for at most a single value of n. For this value of n an A_n and C_n may be found, but for all other values only $A_n = C_n = 0$ are possible solutions.

The source-free solution to Eqs. (1) and (2) is thus characterized by the appearance of only a single one of the $\psi_n(\underline{r})$ which satisfy Laplace's equation. This does not mean, however, that the reactor fluxes will be proportional to $\psi_n(\underline{r})$. It must be remembered that Eqs. (1) and (2) are time independent and if the criticality condition expressed by Eq. (11) is not satisfied by the smallest of the B_n, then there will exist exponentially increasing time-dependent solutions which will eventually become dominant.

It is also clear on physical grounds that the reactor flux can never be proportional to a single one of the ψ_n except ψ_o. This follows since all of the ψ_n except ψ_o are negative over portions of the reactor volume. The physical requirement that the neutron flux must everywhere be real and positive imposes the condition that the exponentially increasing components must be present with sufficient amplitude to compensate for the negative contributions of the time-independent component. The condition of greatest practical interest, of course, is that for which Eq. (11) is satisfied for the smallest of the B_n.

IV. Solution with Sources Present

Returning to the source problem we note that Eqs. (9) and (10) are inhomogeneous for this case and solutions for A_n and C_n may be found for any set of values for the constants in the equations. The numbers S_n are not unknowns of the problem since the $\psi_n(\underline{r})$ are known, $S(\underline{r})$ is given, and it follows from Eq. (6) that

$$S_n = \frac{\int S(\underline{r})\ \psi_n(\underline{r})\ d\underline{r}}{\int \psi_n^2(\underline{r})\ d\underline{r}}, \tag{12}$$

where the integrals extend over the entire volume of the reactor.

It is not difficult to show that Eqs. (9) and (10) yield

$$A_n = \frac{S_n}{\Sigma_1(1 + B_n^2\tau)(1 - k_n)} \qquad (13)$$

and

$$C_n = \frac{k_n}{(1 - k_n)} \cdot \frac{p\ S_n}{\Sigma_2[k_2 + (1 + L^2B_n^2)\ k_1]}, \qquad (14)$$

where

$$k_n = \frac{k_2}{(1 + L^2B_n^2)(1 + \tau B_n^2)} + \frac{k_1}{(1 + \tau B_n^2)}\,. \qquad (15)$$

Equations (12), (13), and (14) provide all of the necessary information from which the fluxes in both groups may be obtained in terms of the functions $\psi_n(\underline{r})$.

It is interesting to note that whenever any of the k_n equal unity then the corresponding A_n and C_n are infinite and will, in effect, be the only significant components in the solution. This is, of course, merely another appearance of the situation described in Sec. III which arises when the reactor is critical (Eq. 11 satisfied) for a particular one of the B_n.

A comparison of Eqs. (13) and (14) shows that more components (modes) will be present in the epithermal than in the thermal flux distribution if thermal leakage is significant. In fact, the ratio of A_n and C_n is just

$$\frac{A_n}{C_n} = \frac{\Sigma_2}{p\Sigma_1}(1 + L^2B_n). \qquad (16)$$

In practice it has been found that about 15 modes may suffice to reproduce the thermal flux distribution to a few tenths of a percent, but as many as 80 modes may be required to reproduce the epithermal flux near the source.

It should be noted that Eqs. (12), (13), and (14) apply in general and are not restricted to a bare reactor. In this case both positive and negative values for B_n^2 will appear and the functions $\psi_n(\underline{r})$ will themselves be linear combinations of the allowable solutions for each root.

In discussing the solution for this case it is convenient to refer to the discussion given by Glasstone and Edlund[1] (pp. 240-247) of the critical-reflected reactor. The functions $\psi_n(\underline{r})$ are obtained for each mode by an exactly analogous procedure as that presented by those authors. Their Eq. (8.55.1) will contain values of B_n in the core as parameters and the successive roots of this equation are the eigenvalues of Eq. (3). No conditions analogous to their Eqs. (8.45.2) and (8.45.3) will be applied. As each B_n is obtained, their equations at the bottom of page 245 may be used to obtain the coefficients A, C, F, and G which then completely specify that ψ_n over the entire reactor volume. By this procedure it is straightforward, though tedious, to solve the source problem in any geometry.

V. A Computer Program for Practical Calculations

A program for the IBM-7090, designated CYS-1, has been written which solves Eqs. (12), (13), and (14) for a cylindrical or annular source in a bare cylindrical reactor and computes the thermal and epithermal fluxes to a prescribed convergence with a maximum of 200 modes. As indicated in the description of input data, the calculation requires values for τ, L^2, Σ_1, Σ_2, $\nu\Sigma_1^f$, $\nu\Sigma_2^f$, and p, together with the core radius and inner and outer radii of the source. In terms of these, the program computes k_1 and k_2 from

$$k_1 = \nu\Sigma_1^f/\Sigma_1 \quad \text{and} \quad k_2 = \nu\Sigma_2^f \frac{p}{\Sigma_2} .$$

The calculations assume that $S(\underline{r})$ is constant and equal to unity (per cm^3-sec).

In cylindrical geometry, for an azimuthally symmetrical core, the functions $\psi_n(\underline{r})$ are simply $J_o(B_n r)$ where the sequence of eigenvalues B_n are determined by the condition that $J_o(B_n R) = 0$, where R is the extrapolated radius of the core. The numbers S_n in Eq. (12) are, for this case,

$$S_n = \frac{2\left\{R_2 J_1(B_n R_2) - R_1 J_1(B_n R_1)\right\}}{B_n \left\{R J_1(B_n R)\right\}^2} \tag{17}$$

since both integrals are elementary (R_1 and R_2 are the inner and outer radii of the source region).

VI. Input and Operation

Program CYS-1 is programmed to be run on the 7090 Monitor System and requires no tapes other than the standard (10, INPUT) and (9, ØUTPUT). Three data cards per case are required and as many cases as desired may be run successively by stacking the input cards.

Card 1. Column 1 must contain a 1-punch. Columns 2-72 may contain any alphameric information and will appear at the top of each page of output.

Card 2. Seven ten digit fields (7E10.4) specify in succession values for τ, L^2, Σ_1, Σ_2, $\nu\Sigma_1^f$, $\nu\Sigma_2^f$, and p.

Card 3. Five ten digit fields (5E10.4) specify in succession values for

a. R_1, inner radius of source region, cm.
b. R_2, outer radius of source region, cm.
c. R, extrapolated radius of core, cm.
d. NR, the number of equally spaced points from $r = 0$ to $r = R$ at which output fluxes are desired. We must have NR $\leqq$ 1000. If NR $\leqq$ 180 and is divisible by four the most orderly form for the output is obtained.
e. CONV, the degree of convergence desired. The contribution of successive modes (ϕ_n) is computed until either

$$\frac{\phi_N}{\sum_{j=1}^{N} \phi_j} \leqq \text{CONV}$$

at every point for each group, or until the maximum of 200 modes has been accumulated.

At the completion of the calculation for each mode the contribution to the flux from all modes up to that point are accumulated and printed out at each space point requested. In addition, values for S_n, k_n, A_n, and C_n are printed out for each mode.

A sample data sheet and several representative pages of output are given in the Appendix.

References

[1] S. Glasstone and M. C. Edlund, The Elements of Nuclear Reactor Theory, D. Van Nostrand Co., Inc., New York, 1952.

[2] A. M. Weinberg and E. P. Wigner, The Physical Theory of Neutron Chain Reactors, University of Chicago Press, Chicago, 1958.

[3] R. L. Murray, Nuclear Reactor Theory, Prentice-Hall, Inc., Englewood Clifts, New Jersey, 1957.

APPENDIX

The following pages present the data sheet for a sample calculation and four typical pages of output for this problem. Convergence to the degree specified was achieved after the inclusion of 78 modes.

TEN DIGIT DATA FORM

PROBLEM CYS-1 Sample Input

REQUEST

CODER C. A. Preskitt

DATE

PAGE of

1	9	11	19	21	29	31	39	41	49	51	59	61	69	71	73 80
1 SAMPLE CASE		FØR REPØRT													
360.0		140.0		0.04		0.007		0.012		0.008		0.7			
0.0		1.0		87.0		88.0		0.005							

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SAMPLE CASE FOR REPORT

MODE NUMBER    1     MODE MULTIPLICATION FACTOR 8.0208E-01

SOURCE AMPLITUDE 4.9016E-04

FAST AMPLITUDE 4.8556E-02    SLOW AMPLITUDE 1.7546E-01


MODE FLUXES

    R       PHI1       PHI2          R       PHI1       PHI2          R       PHI1       PHI2          R       PHI1       PHI2

   0.    4.856E-02 1.755E-01    22.00 4.417E-02 1.596E-01    44.00 3.219E-02 1.163E-01    66.00 1.582E-02 5.717E-02
   1.00 4.855E-02 1.754E-01    23.00 4.377E-02 1.582E-01    45.00 3.151E-02 1.139E-01    67.00 1.504E-02 5.435E-02
   2.00 4.852E-02 1.753E-01    24.00 4.336E-02 1.567E-01    46.00 3.083E-02 1.114E-01    68.00 1.426E-02 5.152E-02
   3.00 4.847E-02 1.752E-01    25.00 4.293E-02 1.551E-01    47.00 3.013E-02 1.089E-01    69.00 1.348E-02 4.871E-02
   4.00 4.841E-02 1.749E-01    26.00 4.249E-02 1.535E-01    48.00 2.943E-02 1.063E-01    70.00 1.270E-02 4.589E-02
   5.00 4.832E-02 1.746E-01    27.00 4.203E-02 1.519E-01    49.00 2.871E-02 1.038E-01    71.00 1.192E-02 4.308E-02
   6.00 4.822E-02 1.743E-01    28.00 4.155E-02 1.501E-01    50.00 2.799E-02 1.012E-01    72.00 1.115E-02 4.028E-02
   7.00 4.810E-02 1.738E-01    29.00 4.106E-02 1.484E-01    51.00 2.727E-02 9.853E-02    73.00 1.037E-02 3.748E-02
   8.00 4.796E-02 1.733E-01    30.00 4.056E-02 1.466E-01    52.00 2.654E-02 9.588E-02    74.00 9.601E-03 3.469E-02
   9.00 4.781E-02 1.728E-01    31.00 4.004E-02 1.447E-01    53.00 2.580E-02 9.321E-02    75.00 8.833E-03 3.192E-02
  10.00 4.763E-02 1.721E-01    32.00 3.951E-02 1.428E-01    54.00 2.505E-02 9.052E-02    76.00 8.069E-03 2.916E-02
  11.00 4.744E-02 1.714E-01    33.00 3.897E-02 1.408E-01    55.00 2.430E-02 8.781E-02    77.00 7.308E-03 2.641E-02
  12.00 4.723E-02 1.707E-01    34.00 3.841E-02 1.388E-01    56.00 2.355E-02 8.508E-02    78.00 6.552E-03 2.367E-02
  13.00 4.700E-02 1.698E-01    35.00 3.784E-02 1.367E-01    57.00 2.279E-02 8.234E-02    79.00 5.800E-03 2.096E-02
  14.00 4.676E-02 1.689E-01    36.00 3.726E-02 1.346E-01    58.00 2.202E-02 7.958E-02    80.00 5.053E-03 1.826E-02
  15.00 4.649E-02 1.680E-01    37.00 3.667E-02 1.325E-01    59.00 2.126E-02 7.681E-02    81.00 4.312E-03 1.558E-02
  16.00 4.621E-02 1.670E-01    38.00 3.606E-02 1.303E-01    60.00 2.049E-02 7.403E-02    82.00 3.577E-03 1.292E-02
  17.00 4.591E-02 1.659E-01    39.00 3.544E-02 1.281E-01    61.00 1.971E-02 7.123E-02    83.00 2.848E-03 1.029E-02
  18.00 4.560E-02 1.648E-01    40.00 3.481E-02 1.258E-01    62.00 1.894E-02 6.843E-02    84.00 2.125E-03 7.678E-03
  19.00 4.527E-02 1.636E-01    41.00 3.417E-02 1.235E-01    63.00 1.816E-02 6.562E-02    85.00 1.409E-03 5.092E-03
  20.00 4.492E-02 1.623E-01    42.00 3.352E-02 1.211E-01    64.00 1.738E-02 6.281E-02    86.00 7.009E-04 2.533E-03
  21.00 4.455E-02 1.610E-01    43.00 3.286E-02 1.187E-01    65.00 1.660E-02 5.999E-02    87.00 1.412E-07 5.103E-07
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SAMPLE CASE FOR REPORT

MODE NUMBER  12    MODE MULTIPLICATION FACTOR 5.0215E-03

SOURCE AMPLITUDE 7.4896E-03

FAST AMPLITUDE 2.8590E-03    SLOW AMPLITUDE 4.3635E-04


MODE FLUXES

   R      PHI1       PHI2          R      PHI1       PHI2          R      PHI1       PHI2          R      PHI1       PHI2

  0.    1.213E-01 2.638E-01     22.00 5.081E-02 1.822E-01     44.00 2.692E-02 1.037E-01     66.00 1.147E-02 4.455E-02
  1.00 1.205E-01 2.634E-01     23.00 4.901E-02 1.780E-01     45.00 2.626E-02 1.007E-01     67.00 1.078E-02 4.218E-02
  2.00 1.181E-01 2.622E-01     24.00 4.728E-02 1.739E-01     46.00 2.559E-02 9.773E-02     68.00 1.011E-02 3.983E-02
  3.00 1.143E-01 2.603E-01     25.00 4.568E-02 1.699E-01     47.00 2.489E-02 9.477E-02     69.00 9.482E-03 3.752E-02
  4.00 1.094E-01 2.578E-01     26.00 4.425E-02 1.659E-01     48.00 2.414E-02 9.185E-02     70.00 8.903E-03 3.524E-02
  5.00 1.037E-01 2.547E-01     27.00 4.300E-02 1.620E-01     49.00 2.334E-02 8.896E-02     71.00 8.367E-03 3.298E-02
  6.00 9.759E-02 2.511E-01     28.00 4.191E-02 1.582E-01     50.00 2.251E-02 8.609E-02     72.00 7.864E-03 3.076E-02
  7.00 9.138E-02 2.471E-01     29.00 4.096E-02 1.545E-01     51.00 2.165E-02 8.325E-02     73.00 7.380E-03 2.856E-02
  8.00 8.542E-02 2.429E-01     30.00 4.009E-02 1.508E-01     52.00 2.079E-02 8.044E-02     74.00 6.898E-03 2.638E-02
  9.00 7.996E-02 2.385E-01     31.00 3.924E-02 1.471E-01     53.00 1.996E-02 7.767E-02     75.00 6.401E-03 2.423E-02
 10.00 7.518E-02 2.340E-01     32.00 3.838E-02 1.436E-01     54.00 1.917E-02 7.494E-02     76.00 5.879E-03 2.209E-02
 11.00 7.116E-02 2.295E-01     33.00 3.747E-02 1.400E-01     55.00 1.844E-02 7.224E-02     77.00 5.326E-03 1.997E-02
 12.00 6.790E-02 2.251E-01     34.00 3.648E-02 1.365E-01     56.00 1.777E-02 6.958E-02     78.00 4.744E-03 1.787E-02
 13.00 6.531E-02 2.206E-01     35.00 3.543E-02 1.330E-01     57.00 1.715E-02 6.695E-02     79.00 4.142E-03 1.579E-02
 14.00 6.328E-02 2.162E-01     36.00 3.433E-02 1.296E-01     58.00 1.657E-02 6.436E-02     80.00 3.533E-03 1.373E-02
 15.00 6.163E-02 2.119E-01     37.00 3.320E-02 1.262E-01     59.00 1.601E-02 6.180E-02     81.00 2.932E-03 1.169E-02
 16.00 6.021E-02 2.076E-01     38.00 3.209E-02 1.228E-01     60.00 1.544E-02 5.927E-02     82.00 2.355E-03 9.676E-03
 17.00 5.886E-02 2.033E-01     39.00 3.103E-02 1.195E-01     61.00 1.486E-02 5.676E-02     83.00 1.812E-03 7.689E-03
 18.00 5.748E-02 1.990E-01     40.00 3.004E-02 1.162E-01     62.00 1.424E-02 5.428E-02     84.00 1.309E-03 5.728E-03
 19.00 5.598E-02 1.948E-01     41.00 2.915E-02 1.130E-01     63.00 1.358E-02 5.181E-02     85.00 8.453E-04 3.794E-03
 20.00 5.436E-02 1.905E-01     42.00 2.834E-02 1.099E-01     64.00 1.290E-02 4.937E-02     86.00 4.130E-04 1.885E-03
 21.00 5.262E-02 1.863E-01     43.00 2.761E-02 1.068E-01     65.00 1.219E-02 4.694E-02     87.00 1.250E-07 5.053E-07
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SAMPLE CASE FOR REPORT

MODE NUMBER  40    MODE MULTIPLICATION FACTOR 4.0764E-04

SOURCE AMPLITUDE 1.9791E-02

FAST AMPLITUDE 6.6641E-04    SLOW AMPLITUDE 9.2094E-06


MODE FLUXES

    R      PHI1       PHI2          R      PHI1       PHI2          R      PHI1       PHI2          R      PHI1       PHI2

   0.    1.585E-01 2.659E-01     22.00 5.026E-02 1.821E-01     44.00 2.702E-02 1.037E-01     66.00 1.152E-02 4.455E-02
   1.00  1.509E-01 2.653E-01     23.00 4.896E-02 1.780E-01     45.00 2.624E-02 1.007E-01     67.00 1.091E-02 4.219E-02
   2.00  1.335E-01 2.635E-01     24.00 4.754E-02 1.739E-01     46.00 2.539E-02 9.770E-02     68.00 1.027E-02 3.786E-02
   3.00  1.161E-01 2.609E-01     25.00 4.609E-02 1.699E-01     47.00 2.456E-02 9.474E-02     69.00 9.664E-03 3.755E-02
   4.00  1.048E-01 2.578E-01     26.00 4.483E-02 1.660E-01     48.00 2.382E-02 9.181E-02     70.00 9.109E-03 3.526E-02
   5.00  9.858E-02 2.543E-01     27.00 4.372E-02 1.621E-01     49.00 2.311E-02 8.892E-02     71.00 8.540E-03 3.301E-02
   6.00  9.332E-02 2.506E-01     28.00 4.254E-02 1.583E-01     50.00 2.233E-02 8.606E-02     72.00 7.932E-03 3.077E-02
   7.00  8.755E-02 2.467E-01     29.00 4.128E-02 1.545E-01     51.00 2.154E-02 8.324E-02     73.00 7.340E-03 2.856E-02
   8.00  8.249E-02 2.425E-01     30.00 4.012E-02 1.508E-01     52.00 2.082E-02 8.045E-02     74.00 6.800E-03 2.637E-02
   9.00  7.892E-02 2.383E-01     31.00 3.912E-02 1.471E-01     53.00 2.014E-02 7.769E-02     75.00 6.268E-03 2.421E-02
  10.00  7.598E-02 2.340E-01     32.00 3.811E-02 1.435E-01     54.00 1.943E-02 7.497E-02     76.00 5.697E-03 2.206E-02
  11.00  7.275E-02 2.297E-01     33.00 3.701E-02 1.399E-01     55.00 1.868E-02 7.227E-02     77.00 5.121E-03 1.995E-02
  12.00  6.952E-02 2.252E-01     34.00 3.594E-02 1.364E-01     56.00 1.797E-02 6.961E-02     78.00 4.591E-03 1.785E-02
  13.00  6.696E-02 2.208E-01     35.00 3.500E-02 1.330E-01     57.00 1.733E-02 6.698E-02     79.00 4.088E-03 1.578E-02
  14.00  6.490E-02 2.164E-01     36.00 3.411E-02 1.295E-01     58.00 1.667E-02 6.437E-02     80.00 3.557E-03 1.373E-02
  15.00  6.274E-02 2.120E-01     37.00 3.314E-02 1.262E-01     59.00 1.597E-02 6.180E-02     81.00 3.001E-03 1.170E-02
  16.00  6.042E-02 2.076E-01     38.00 3.214E-02 1.228E-01     60.00 1.528E-02 5.925E-02     82.00 2.480E-03 9.696E-03
  17.00  5.838E-02 2.033E-01     39.00 3.125E-02 1.195E-01     61.00 1.465E-02 5.673E-02     83.00 1.999E-03 7.714E-03
  18.00  5.674E-02 1.989E-01     40.00 3.043E-02 1.163E-01     62.00 1.404E-02 5.424E-02     84.00 1.505E-03 5.753E-03
  19.00  5.514E-02 1.947E-01     41.00 2.957E-02 1.131E-01     63.00 1.338E-02 5.178E-02     85.00 9.777E-04 3.813E-03
  20.00  5.336E-02 1.904E-01     42.00 2.865E-02 1.099E-01     64.00 1.271E-02 4.934E-02     86.00 4.657E-04 1.896E-03
  21.00  5.166E-02 1.862E-01     43.00 2.779E-02 1.068E-01     65.00 1.210E-02 4.693E-02     87.00 1.255E-07 5.052E-07
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SAMPLE CASE FOR REPORT

MODE NUMBER  78    MODE MULTIPLICATION FACTOR 1.0594E-04

SOURCE AMPLITUDE 1.4704E-02

FAST AMPLITUDE 1.2951E-04    SLOW AMPLITUDE 4.6900E-07


MODE FLUXES

   R       PHI1       PHI2         R       PHI1       PHI2         R       PHI1       PHI2         R       PHI1       PHI2

  0.     1.712E-01 2.660E-01    22.00 5.027E-02 1.821E-01    44.00 2.700E-02 1.037E-01    66.00 1.150E-02 4.455E-02
  1.00  1.541E-01 2.653E-01    23.00 4.888E-02 1.780E-01    45.00 2.620E-02 1.007E-01    67.00 1.089E-02 4.219E-02
  2.00  1.296E-01 2.634E-01    24.00 4.748E-02 1.739E-01    46.00 2.539E-02 9.770E-02    68.00 1.029E-02 3.986E-02
  3.00  1.163E-01 2.607E-01    25.00 4.617E-02 1.700E-01    47.00 2.461E-02 9.474E-02    69.00 9.686E-03 3.755E-02
  4.00  1.061E-01 2.578E-01    26.00 4.489E-02 1.660E-01    48.00 2.383E-02 9.181E-02    70.00 9.101E-03 3.526E-02
  5.00  9.873E-02 2.543E-01    27.00 4.366E-02 1.621E-01    49.00 2.306E-02 8.892E-02    71.00 8.511E-03 3.300E-02
  6.00  9.252E-02 2.506E-01    28.00 4.247E-02 1.583E-01    50.00 2.232E-02 8.606E-02    72.00 7.941E-03 3.077E-02
  7.00  8.739E-02 2.467E-01    29.00 4.131E-02 1.545E-01    51.00 2.157E-02 8.324E-02    73.00 7.362E-03 2.856E-02
  8.00  8.301E-02 2.426E-01    30.00 4.020E-02 1.508E-01    52.00 2.084E-02 8.045E-02    74.00 6.804E-03 2.637E-02
  9.00  7.906E-02 2.383E-01    31.00 3.907E-02 1.471E-01    53.00 2.011E-02 7.769E-02    75.00 6.240E-03 2.421E-02
 10.00  7.570E-02 2.340E-01    32.00 3.805E-02 1.435E-01    54.00 1.941E-02 7.497E-02    76.00 5.691E-03 2.206E-02
 11.00  7.251E-02 2.297E-01    33.00 3.700E-02 1.399E-01    55.00 1.870E-02 7.227E-02    77.00 5.144E-03 1.995E-02
 12.00  6.977E-02 2.253E-01    34.00 3.601E-02 1.364E-01    56.00 1.801E-02 6.961E-02    78.00 4.602E-03 1.785E-02
 13.00  6.712E-02 2.208E-01    35.00 3.501E-02 1.330E-01    57.00 1.732E-02 6.698E-02    79.00 4.071E-03 1.578E-02
 14.00  6.479E-02 2.164E-01    36.00 3.406E-02 1.295E-01    58.00 1.664E-02 6.437E-02    80.00 3.537E-03 1.373E-02
 15.00  6.254E-02 2.120E-01    37.00 3.311E-02 1.262E-01    59.00 1.597E-02 6.180E-02    81.00 3.020E-03 1.170E-02
 16.00  6.049E-02 2.076E-01    38.00 3.217E-02 1.228E-01    60.00 1.531E-02 5.925E-02    82.00 2.497E-03 9.696E-03
 17.00  5.855E-02 2.033E-01    39.00 3.129E-02 1.195E-01    61.00 1.466E-02 5.673E-02    83.00 1.992E-03 7.713E-03
 18.00  5.671E-02 1.989E-01    40.00 3.039E-02 1.163E-01    62.00 1.401E-02 5.424E-02    84.00 1.480E-03 5.752E-03
 19.00  5.500E-02 1.947E-01    41.00 2.953E-02 1.131E-01    63.00 1.338E-02 5.178E-02    85.00 9.850E-04 3.813E-03
 20.00  5.333E-02 1.904E-01    42.00 2.866E-02 1.099E-01    64.00 1.274E-02 4.934E-02    86.00 4.878E-04 1.896E-03
 21.00  5.180E-02 1.862E-01    43.00 2.784E-02 1.068E-01    65.00 1.212E-02 4.693E-02    87.00 1.255E-07 5.052E-07
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