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A Review of Stirling Engine Mathematical Models

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ABSTRACT

As requested by the Department of Energy, a review of existing mathematical models for Stirling engine thermodynamic analysis has been performed. Twenty-five models were identified through extensive literature search; 19 of these were published in sufficient detail for review. Each individual model's assumptions, limitations, predictability, and applicability were assessed by using a two-part review format consisting of model description and validation. According to their design methods, models were grouped into four categories by degree of sophistication: approximate (first-order) methods, decoupled (second-order) methods, nodal (third-order) analyses, and method of characteristics. The salient characteristics of the models were summarized in two tables for cross-reference.

In the course of this review, these points were established.

1. Utilization of a detailed design method does not ensure enhanced model performance. There is no evidence that the existing third-order analyses are superior to the second-order methods.

2. Model validation is largely limited to kinematic engines with emphasis on thermodynamic analysis. With increasingly important applications for free-piston Stirling engines, it is highly recommended that dynamic analysis should be integrated into thermodynamic study in future modeling efforts.

3. To achieve an in-depth evaluation of the individual model's assumptions and validation would require model acquisition and more abundant experimental data than were available for this review.

4. The ranking of the various models is not possible by this review. Only when all models can be run with a common set of input data and compared with well-defined experimental data can a fair or valid comparison be made.

1. INTRODUCTION

As requested by the Department of Energy (DOE), screening of the existing computer programs for Stirling engine analysis has been performed. This report is intended to provide a user guide for quick reference to the existing computer codes. Only those programs related to thermodynamic analysis were reviewed. Although dynamic analysis is critical to free-piston Stirling engine studies, it will not be discussed in this report.

Among the existing computer programs identified (25 total), some 19 were published in sufficient detail for reviewing; of these, 10 were already evaluated, most extensively by Martini,[1–4] and to a lesser extent by Urieli[5] and Walker.[6] We will conduct a state-of-the-art review through an independent assessment, even though in the process of reviewing, some degree of overlap among Oak Ridge National Laboratory (ORNL), Martini, Urieli, and Walker is inevitable.

Those models reviewed were grouped by their engine design methods and basic assumptions for cycle analysis. Four distinct methods were identified: approximate (first-order), decoupled (second-order), nodal (third-order), and method of characteristics. First-order methods are good for back-of-the-envelope evaluations. Second-order analyses are good for interactive design and optimization. Third-order methods are very detailed and can be used to simulate engine operation in a way that would be difficult if not impossible to measure experimentally. The method of characteristics is based on the theories of gas dynamics. These design methods are defined and reviewed in Sect. 2.

In Sect. 3, models will be reviewed one by one according to a predevised format containing two major parts: model description and model validation. Model description will further discuss assumptions and limitations; model validation includes predictability and application. Comments on individual model performance are provided wherever possible. In Sect. 4, two extensive tables that summarize the significant features of the models are presented for cross-reference. The tables include information such as principal investigator, affiliation, model classification, code listing availability, model validation, and references. Table 1 summarizes the features of seven second-order models, and Table 2 includes information about ten third-order models.

Conclusions with recommendations are furnished in Sect. 5. Finally, a bibliography essential to model review is attached for further studies.

2. REVIEW OF DESIGN METHODS

The four identified engine design methods are first-order, second-order, third-order, and the method of characteristics. The definitions given below are similar to those of Martini[3] and Organ.[7,8]

2.1 First-Order or Approximate Design Methods

First-order design methods are used for back-of-the-envelope Stirling engine performance predictions. Calculation of power output starts with an ideal loss-free analysis, such as the Schmidt equation published by Martini[3] or the generalized Beale number derived by Senft.[9] A simple correction factor is then used to find the brake power output from the ideal power output. Similarly, brake efficiency is usually computed from a corrected Carnot efficiency. The corrections for all of the various losses in a Stirling engine are consolidated into generalized correction factors. These efficiency and power correction factors are determined from experience with real engines. For example, most well-designed Stirling engines achieve a brake efficiency that is 50 to 70% of the Carnot value. First-order analyses provide a quick way to estimate the relationship between the overall size of an engine and its power output, but they are not very useful as detailed design tools for Stirling engines.

2.2 Second-Order or Decoupled Design Methods

This design method begins with a simplified cycle analysis to determine a basic power output and heat input. Various power losses are then subtracted from the basic power output, and heat losses are added to the heat input to arrive at a net performance prediction. The major improvement of the second-order methods relative to the first-order design methods is that individual loss mechanisms are identified and quantified. Power losses can include fluid and mechanical friction, transient heat transfer (hysteresis) losses in cylinders, and gas leakage past piston seals. Heat losses include displacer shuttle losses, wall conduction, and imperfect heat transfer in regenerators. In all second-order design methods, it is assumed that the energy losses are not dependent on each other; that is, they are decoupled.

Second-order design methods may be further subdivided into three categories according to the way the variable gas volumes are handled in the simplified cycle analysis: isothermal, adiabatic, and semi-adiabatic. These terms were derived according to heat transfer rate between gas spaces and engine cylinders. If the rate is infinite, it is isothermal. On the other hand, if the rate is zero, it is adiabatic. Semi-adiabatic is the process somewhere in between with a limited heat transfer rate.

2.2.1 Isothermal analysis

This analysis is based on the classical Schmidt isothermal cycle, which, by allowing for sinusoidal volume variations, is a slightly more realistic form of the ideal Stirling cycle. All gas in the expansion space is maintained at the heat source temperature, and all gas in the compression space is maintained at the heat sink temperature because infinite heat transfer coefficients are assumed. Perfect regeneration is also assumed (i.e., the local gas temperature is equal to the local wall temperature in the regenerator, and there is no axial heat conduction). All heat input to the isothermal cycle occurs in the expansion space, and all heat output occurs in the compression space. A simple closed form solution exists for the Schmidt cycle.

2.2.2 Adiabatic analysis

The adiabatic cycle assumes that the compression and expansion spaces are perfectly insulated. All heat input to the cycle occurs in the heater, and all heat output occurs in the cooler. Gases leave the heater at the heat source temperature and are mixed perfectly as soon as they enter the expansion space. Similarly, gases leave the cooler at the heat sink temperature and are mixed perfectly as soon as they enter the compression space. Again, perfect regeneration is assumed. The adiabatic cycle is a more realistic simplification of a Stirling engine than the Schmidt cycle, especially for large engines operating at high frequencies. However, an isothermal second-order analysis can be just as accurate as an adiabatic second-order analysis as long as proper adiabatic loss terms are subtracted from the isothermal cycle predictions. Solution of the adiabatic cycle requires a simple numerical integration.

2.2.3 Semi-adiabatic analysis

Semi-adiabatic cycles allow for nonzero, finite heat transfer coefficients. The simplest semi-adiabatic cycle, first analyzed by Finkelstein,[10] accounts for heat transfer in the expansion and compression spaces. The wall temperatures of these volumes are assumed to be constant with respect to time and equal to the heat source and heat sink temperatures, respectively. The heater, cooler, and regenerator are assumed to behave perfectly. This semi-adiabatic cycle can actually result in efficiency predictions that are lower than either the purely adiabatic cycle or the isothermal cycle. This is caused by irreversible heat transfer losses across the temperature difference between the gas and the cylinder walls in the compression and expansion spaces. Solution of this semi-adiabatic cycle requires a simple numerical integration.

2.3 Third-Order or Nodal Design Methods

Third-order design methods, also known as nodal analyses, consist of three basic procedures: (1) divide the engine into a network of nodes or control volumes; (2) set up the differential equations for conservation of mass, momentum, and energy, plus equation of state for the working gas; and (3) solve simultaneously the system of difference equations by some adequate numerical method. There are two subclasses under this method: one, most rigorous, and the other, less rigorous. The rigorous third-order analyses solve all the equations except for the use of steady flow correlations for heat transfer and friction flow because no correlations of universal validity exist for unsteady flow in today's technology. The less rigorous third-order models simplify the numerical computations by omitting some of the terms from the governing differential equations. It is assumed that certain losses can be decoupled from the main calculation to improve the speed of computations. There are three common simplifications: (1) inertial terms are ignored in the momentum equation, but flow friction terms are retained; (2) both inertial and flow friction terms are ignored, that is, the momentum equation is not used and a uniform pressure is assumed throughout the engine; and (3) kinetic energy terms are ignored in the energy equation.

All of the nodal design methods use finite differencing of the spatial derivatives to convert the partial differential equations to a system of ordinary differential equations (with only time derivatives remaining). Each conservation equation is represented by a difference equation at each node. The numerical methods for solving this system of ordinary differential equations are divided into two categories: explicit (forward-differencing) and implicit (backward-differencing) techniques. In the explicit integrations, the thermodynamic information (such as pressure and temperature) at a new time is computed from time derivatives that were evaluated at the previous time. The simplest explicit method is the Euler method, although more accurate techniques, such as the Runge-Kutta method, may be used. Explicit techniques are sometimes plagued by numerical oscillations and instabilities, especially if time steps are too large. In contrast, an implicit integration is always numerically stable. The implicit method solves the system of ordinary differential equations by computing the thermodynamic information at a new time from time derivatives that are evaluated at the new time. A large matrix must be inverted at each time step. Because of the lack of numerical instabilities, implicit integrations can use larger time steps. This reduces computer execution times, but it may also reduce the accuracy of the numerical approximation.

Third-order design methods attempt to consider the many different complex processes coexisting in a Stirling engine. It is hypothesized that the various processes assumed to be decoupled in the second-order design methods do in reality significantly interact. Whether this assumption is true remains to be seen after further theoretical and experimental studies. The third-order methods are the most sophisticated, and by far the most expensive in computer time; but there is no evidence that they give the best results. In fact, the results from second-order codes are at least as good when compared with experimental data. Furthermore, some workers have questioned the mathematical foundation of the third-order methods: it is believed that under certain circumstances the solutions may converge to values that are mathematically and computationally stable, but do not correspond to a real physical state. In any case, more experimental data are required for a fair assessment.

2.4 Method of Characteristics

The method of characteristics solves systems of nonlinear partial differential equations of hyperbolic type by determining the characteristic curves for the equations. The characteristic curves are used to transform the partial differential equations into a system of ordinary differential equations that are valid only along the characteristic curves. This method has been used successfully in the study of compressible gas flow and has been applied to the analysis of one-dimensional, unsteady flow in Stirling engines.

In one-dimensional, unsteady flow, the characteristic curves are in the position-time plane on which the partial derivatives with respect to position and time of the fluid properties (such as density, velocity, and temperature) are indeterminate and may, therefore, undergo arbitrary discontinuities. To establish the conditions for indeterminacies, the conservation equations (mass, momentum, energy) along with the total differentials of the fluid properties are expressed in matrix notation with the partial derivatives of the fluid properties as the dependent variables. The characteristic curves are found by setting the determinant of the coefficient matrix equal to zero. For more information on the theory and applications of the method of characteristics, readers should refer to excellent books by Shapiro[11] and Leipmann and Roshko.[12]

The method of characteristics can be applied at different levels of complexity to Stirling engine analyses. In rigorous analyses, all three conservation equations are solved simultaneously. In approximate analyses, however, some simplifying assumptions are used to solve one of the conservation equations independently. The two remaining conservation equations are then solved simultaneously by the method of characteristics.

3. REVIEW OF MODELS

Assessments of the 19 models reviewed are presented. The reviewing methodology consists of model description and validation. The model descriptions present further their basic assumptions and limitations; model validation discusses predictability and applicability. Grouping of models fell naturally into the three major design methods: seven second-order, ten third-order, and two methods of characteristics.

Six other Stirling engine computer models were identified, but not reviewed. Rauch[13] has described a model that is based on a second-order design method. Berggren[14] and Andersen[15] have developed models that utilize third-order design methods. Sirett[16] has developed a model that solves the complete set of conservation equations using the method of characteristics. Vincent et al.[17] of Energy Research and Generation, Inc., give a brief description of a thermodynamic model, but insufficient details are provided to allow a review. Models have also been developed at Harwell,[18] and some are thought to exist at Philips, but the authors of this report have not acquired any documentation that describes these models.

3.1 Second-Order Design Methods

There are seven models in this category: one isothermal, five adiabatic, and one semi-adiabatic.

3.1.1 Model by Martini (1978) – isothermal analysis

Martini[1,3] has published detailed documentation of his second-order model. In his analysis, Martini assumed that the time-dependent gas temperatures in the expansion and compression spaces of an actual Stirling engine can be expressed as time-averaged effective temperatures. The effective hot gas temperature will be less than the heater temperature and the effective cold gas temperature greater than the cooler temperature. These temperatures were derived from the computed heat transfer coefficients in both the gas heater and gas cooler as well as from the computed heat requirement. An iterative procedure is needed as described in great detail by Martini.

To validate the model, Martini[3] applied his code to two reference engines, GPU-3 and 4L23, both of General Motors (GM). When compared with the experimental values for the GPU-3 and the values predicted by GM for the 4L23, Martini's calculated power and efficiency were found to be within 20% error bands, if no correction factor for flow resistance is used. Considerable improvement (reducing the error bands by half) can be made if either (1) a correction factor of about 2.9 is applied to the flow resistance coefficients, or (2) the computed heat transfer coefficients are adjusted by a factor of 0.8.

3.1.2 Model by Qvale (1967) - adiabatic analysis

Qvale's[19,20] second-order model is based on an idealized adiabatic cycle that has no friction or seal leakage. The pressure changes, mass variations, piston displacements, and volume changes are all assumed to be sinusoidal. The problem was formulated with pressure, temperature, and mass as the independent variables. Thus, the piston displacements are computed values that depend on the three independent variables. This type of analysis is more suitable for engine synthesis than for performance predictions of a specific engine.

Qvale[19] validated his model by comparing it with the Allison PD-67A experimental Stirling engine. His predictions for heat input, work output, and indicated efficiency compare favorably with the test data over a range of engine speed (1500 to 3000 rpm).

3.1.3 Model by Rios (1969) - adiabatic analysis

Both Rios and Qvale did their graduate work for Professor J. L. Smith at Massachusetts Institute of Technology (MIT). Rios[21] expanded Qvale's work on their adiabatic second-order model. Rios used the same basic assumptions as Qvale but changed the formulation of the problem so that nonsinusoidal piston displacements (such as those resulting from crankshafts with short connecting rods) could be specified.

Rios' graduate work was applied to Stirling refrigerators. However, Martini[3] obtained the Rios computer code and modified it to suit a Stirling engine application. Martini then compared the modified Rios case with 18 data points from the GM 4L23 engine. The code overpredicted brake power and efficiency by an average of 24 and 16%, respectively.

3.1.4 Model by Lee et al. (1981) - adiabatic analysis

The Lee et al.[22] model of Foster Miller Associates is an application of the Rios adiabatic second-order analysis. In this model, a unique power loss mechanism was introduced. This is the cyclic heat transfer loss (or gas spring hysteresis loss) that results from periodic heating and cooling of the working gas near the gas-wall interfaces inside cylinders, manifold spaces, connecting tubes, and reservoirs. In their applications to the Viking-1 engine, Lee et al.[23] were able to quantify the cyclic heat transfer loss. Among the four major power losses identified (adiabatic, cyclic heat transfer, pressure drop, and heat exchanger ΔT), it was shown that cyclic heat transfer loss was ranked second in significance, contributing about one-third of the total power loss.

The Lee et al.[22] model was refined and verified by the Sunpower, Inc., third-order analysis. For model validation, the model compared fairly well to the GPU-3 test data. The model overpredicted the brake power and brake efficiency by <15%, provided that a correction factor of 2.5 was applied to flow resistance.

3.1.5 Models by Shoureshi (1982) - adiabatic and isothermal analyses

With objectives in low temperature-ratio applications, Shoureshi[24] developed two Stirling engine mathematical models. Both models are second-order design methods and are called the complete model and the simplified model.

The complete model is based on Rios' adiabatic second-order analysis. However, updated correlations for two important losses were used: mechanical friction and transient heat transfer losses. For mechanical friction, Shoureshi developed a correlation that is based on internal combustion engine data. For transient heat transfer losses in the cylinders, Shoureshi provided an alternate approach that excluded heat transfer enhancement factors as originally derived by Lee et al. (see Sect. 3.1.4).

To achieve an efficient optimization design method and a closed-form solution, Shoureshi[25] developed a simplified model. This method involves the Schmidt isothermal analysis plus a two-step correction for the net power output and heat input. In the first step, an adiabatic correction (the correction from Schmidt isothermal engine analysis to adiabatic analysis with perfect components) was introduced. The second step involved deductions of all identifiable decoupled losses. These loss terms were similar to the ones used in the adiabatic analysis of the complete model. The procedure of the first step was to derive appropriate corrections for the Schmidt isothermal work output with three factors: corrections for temperature ratio, phase-angle difference between the displacer and the piston, and dead volumes. These factors were obtained by comparing the computed Schmidt work output with that computed from the adiabatic analysis in the complete model, which was assumed to be a reference model by Shoureshi. In addition, a correction for Carnot efficiency was determined as a function of temperature ratio. In the process of the second step, various losses, expressed in closed-form solutions, were further deducted from the basic work output and added to the heat input to obtain the net work output and heat input.

To verify the complete model, Shoureshi[24] compared his predictions with measurements from the following high-temperature engines: Philips, Allison, and GPU-3. It was shown that the complete model predicted engine performance within the range of experimental uncertainty. Similar conclusions were claimed for the simplified model predictions.

3.1.6 Model by Heames (1982) - adiabatic analysis

Heames et al.[26] of Argonne National Laboratory (ANL) developed a user-oriented Stirling engine analysis code. The computer program consists of four modules: (1) input processor, (2) output processor, (3) standard function module, and (4) analysis module. The input and output modules use a flexible format that simplifies data entry and retrieval for many different Stirling engine configurations. The two modules also provide the capability to specify multiple computer executions for parametric studies. The output module saves the parametric results on an external file for users with graphics capabilities. The standard function module is a library of subprograms that provides the user with numerous correlations and functions that are used commonly in Stirling engine analyses. Included in the function module are (1) temperature-dependent correlations for the physical properties of many different fluids and metals; (2) friction factor and heat transfer correlations for many different heater, cooler, and regenerator configurations; (3) engine heat loss correlations such as shuttle heat transfer and cylinder wall conduction; and (4) subroutines to compute cylinder volume variations for different types of crank drive mechanisms. The analysis module contains the Stirling engine thermodynamic computations. A user will eventually be able to select from a number of different thermodynamic algorithms. However, the only analysis method in the present edition of the computer program is based on Rios' adiabatic second-order design method. A tape copy and a user's manual for ANL's Stirling engine design code will be available upon request through the National Energy Software Center.

ANL's computer code has been validated against GPU-3 experimental engine data. The computer predictions for indicated power output compare favorably with the experimental data. However, the computer program appears to overestimate efficiency by as much as five percentage points, especially at low engine speeds.

3.1.7 Model by Feurer (1973) - semi-adiabatic analysis

Philips has published very little about their Stirling engine modeling activities. However, a paper by Feurer[27] of Entwicklungsgruppe Stirlingmotor MAN-MWM (MAN/MWM), a Philips licensee, disclosed their semi-adiabatic second-order analysis. This cycle is an adiabatic cycle that allows for nonzero, finite heat transfer coefficients in the cylinders, regenerator, and heat exchangers. The power output and efficiency are first calculated based on this cycle, then are corrected for: (1) losses due to nonsinusoidal crank motion, (2) residual adiabatic losses that the simplified heat transfer coefficients do not account for, (3) flow friction losses, (4) mechanical friction losses, and (5) static conduction losses.

In his theoretical studies, Feurer showed that all the loss mechanisms are phase-angle dependent. One major conclusion is that the maximum efficiency and the maximum power output do not occur at the same phase angle. Model validation has not been found in the open literature because of proprietary controls.

3.2 Third-Order Design Methods

There are ten models in this category: six use less rigorous methods, three use most rigorous methods, and the simplifications in the final one (model by Zacharias, Sect. 3.2.10) were not stated clearly.

3.2.1 Model by Finkelstein (1975) - less rigorous analysis

As a pioneer in Stirling engine analysis, Finkelstein developed a third-order method in the early 1960s. This review is based on a more recent version published in 1975.

Finkelstein's[28] model is a third-order design method, but less rigorous than that of Urieli (see Sect. 3.2.7), which will be described later. Finkelstein made two major assumptions in his derivation of the governing differential equations. First, the gas kinetic energy term was ignored in the energy equation. Second, the momentum equation was reduced to the form of an equivalent orifice equation.

Finkelstein divided both the engine components and gas spaces into nodal networks. The nodes were treated as regions of variable temperature and mass. Energy and mass transfer between nodes resulted from the computed temperature and pressure differences. All paths for conduction, convection, and mass transport were included. Finkelstein used a special technique to reduce the convergence time for finding a cyclic, steady state, nodal temperature distribution. This was accomplished by manually adjusting the temperatures of each metal node at the end of every piston revolution based on net heat balances for the nodes. For example, if a nodal heat balance across one piston cycle shows that there is a net flow of heat into a metal node, then the temperature of the node is adjusted upward.

Finkelstein reported that his model has been validated, but the results are not available for evaluation. However, his program is now commercially available for general use on the CDC Cybernet computer system.

3.2.2 Model by Tew et al. (1978) - common pressure analysis

The Tew et al.[29,30] model of the National Aeronautics and Space Administration (NASA) Lewis Research Center is a less rigorous third-order design method. They used three basic equations (conservation of mass, energy, and equation of state) to determine the thermodynamics of the gas (temperature and mass distributions and pressure level) at each of 13 nodes. Several simplifications were used to minimize the numerical integration times: (1) the momentum equation was totally ignored and a common pressure throughout the 13 gas nodes was assumed during each time step, (2) kinetic energy was neglected in the energy equation, and (3) the three processes that contribute to gas temperature changes (pressure changes, gas mixing, and heat transfer) were treated independently. The numerical integration required 30 to 40 piston cycles to approach cyclic steady state distributions of gas mass and regenerator metal temperature. Pressure drop, shuttle, and conduction losses were accounted for by including these calculations only during the last cycle of the numerical integration. In the last cycle, the common pressure at each time step was assumed to exist at the center of the regenerator. The compression and expansion space pressures were then calculated by estimating the pressure drops in each control volume from steady state correlations and summing these pressure drops. The net area enclosed by the pressure-volume curves of the compression and expansion spaces is equal to the indicated work output. Thus, the pressure drops are decoupled from the calculation of mass and temperature distributions, but they do affect the work output predictions.

Tew et al.[31] compared their theory with experimental data from the GM GPU-3 Stirling engine. When the regenerator friction factor was increased by factors of 4.0 and 2.6 for hydrogen and helium, respectively, the model overpredicted both brake power and efficiency by 5 to 30%. Tomazic[32] also compared the NASA-Lewis model with experimental power output data from the USS P-40 Stirling engine. For this comparison, measured flow resistances were used in the model rather than computed ones. The predicted brake power values were consistently high for all engine speeds (500 to 4000 rpm) and pressures (4 to 15 MPa).

3.2.3 Model by Giansante (1980) - common pressure analysis

Giansante[33] of Mechanical Technology, Inc. (MTI), acquired NASA-Lewis' code and applied it to the free-piston Stirling test engine built for DOE. Because the NASA-Lewis code was developed specifically for kinematic type engines, the following modifications have been accommodated: (1) gas springs for the free pistons were modeled as separate control volumes with heat transfer and seal leakage, (2) centering ports for the free pistons (these eliminate free-piston migration due to seal leakage) were simulated in a way similar to the NASA-Lewis method, and (3) a subroutine to predict free-piston dynamics was implemented. Two interesting types of load were a linear alternator and a velocity-cubed dissipator.

MTI compared their free-piston Stirling model with two experimental points (high and low powers) from their DOE 1-kW Technology Demonstrator Engine. In the comparison, the piston dynamics (positions, velocities, accelerations) were specified in the code from experimental data. The indicated power and indicated efficiency were overpredicted by an average of 8 and 55%, respectively. Unfortunately, the scope of MTI's validation effort was very limited, and they were not able to identify any specific improvements to the NASA-Lewis assumptions.

3.2.4 Model by Chiu et al. (1979) - less rigorous analysis

Chiu et al.[34] of General Electric Company (GE) developed a thermodynamic program (TDP) that comprises thermodynamic and dynamic analysis for free-piston Stirling engine applications. They (1) discretized the engine into a nodal network, (2) converted the differential equations into difference equations, and (3) applied a transient finite-difference integration scheme. Some key assumptions were the use of steady state empirical correlations for heat transfer and flow resistance, plus the ideal gas law. The current code has not incorporated such important losses as static heat conduction, shuttling losses, and transient heat transfer. These losses were estimated manually and corrected ad hoc. This last aspect has a close resemblance to second-order methods.

GE's model has been validated against a prototype machine with the results being claimed as relatively satisfactory. In fact, for the heat input, predictions and test data agreed reasonably well, within about $\pm$15%. For the indicated power, the model overpredicted by an average of about 41%; and for the indicated efficiency, the error averaged about 37%.

GE's predictions could have perhaps been made to fit the data better, but their policy was to minimize the use of correction factors in the heat transfer and fluid friction correlations.

3.2.5 Model by Azetsu et al. (1982) - less rigorous analysis

The Azetsu et al.[35] model, the only Japanese mathematical model published in English, may be classified as a third-order analysis. However, it is less rigorous than those of Urieli (Sect. 3.2.7) or Schock (Sect. 3.2.8). In many aspects, it is similar to the Tew et al. model, because (1) a common pressure throughout all spaces was assumed, (2) gas kinetic energy was ignored, and (3) the momentum equation was decoupled from the other equations. A steady state momentum equation was used for pressure-drop calculations only. Similarity also has been found in the solution method and numerical convergence procedure. The model was designed to adapt to various engine configurations, operating conditions, and thermal properties.

The Azetsu et al. model was validated by a two-piston Stirling demonstration engine, manufactured and constructed at the University of Tokyo, Japan. Good agreement has been obtained for P-V diagrams, cyclic temperature variation, indicated work, and thermal efficiency. They also concluded that engine performance was influenced significantly by phase angle and dead volume.

3.2.6 Model by Vanderbrug (1977) - less rigorous analysis

Vanderbrug[36] of Jet Propulsion Laboratory (JPL) presented a general purpose program for Stirling engine analysis. The program, known as Stirling Cycle Computer Model (SCCM), was initially designed for Stirling engines in underwater applications. This analysis used a simplified basic equation set in which the gas-inertial effects were ignored. Attributes of the SCCM program include (1) thermodynamic processes for each control volume are quasi static during a small time interval, (2) empirical or theoretical correlations for component performance characteristics are readily modeled by a lumped parametric (nodal) method, and (3) user-oriented subroutines can be assembled for any particular physical systems to be modeled.

Hoehn[37] validated Vanderbrug's model against the single JPL experimental data point published so far. The net indicated power predicted by the SCCM program was only 1% higher than the measured value, even though the predicted magnitudes of indicated power for the individual expansion and compression pistons were underestimated by 15 and 32%, respectively. Hoehn also applied a Schmidt analysis to the single data point. The Schmidt predictions were nearly as good as the SCCM model. Therefore, many more data points are needed for a meaningful evaluation.

3.2.7 Model by Urieli (1977) - rigorous analysis

Urieli's[38] model is a rigorous nodal analysis. He considered the full conservation equations by retaining the kinetic energy and the gas inertia effects. Salient features consist of (1) piecewise approximation, that is, discretizing the engine into control volumes of various sizes and shapes; (2) converting the partial differential equations into a system of ordinary differential equations by transforming all differentials to difference quotients except for the time variable; and (3) solving these ordinary differential equations using the fourth-order Runge-Kutta method with a stationary initial condition. Also, the model applied a convergence scheme that adjusts the matrix temperatures at the end of each cycle in accordance with the net heat transferred in the control volumes. Urieli stated that convergence usually occurs within ten cycles.

The program,[39] written in Fortran language, was fully documented. It is efficient and very versatile. Urieli has shown that a minimal number of cells (about 33) can be used satisfactorily to find the performance of a particular machine, with a corresponding saving in computer time. Also, three-dimensional plots showing the behavior of the temperature, flow, and pressure profiles through the cycle are possible, thus helping to provide further insight into the detailed behavior of Stirling cycle machines.

Urieli's model has been validated at three engine operating frequencies of the University of Witwatersrand (South Africa) test engine.[40] When compared with the experimental data, the model underpredicted the heat transfer rates in the heater and cooler and the power output by averages of 7, 13, and 40%, respectively. However, it is important to note that these predictions are for experimental cases that produce very little net power. Thus, errors in the net power output predictions that are small compared with the heat input may appear rather large relative to the small magnitude of the net power output.

3.2.8 Model by Schock (1978) - rigorous analysis

Schock of Fairchild Industries developed a Stirling Nodal Analysis Program (SNAP), but full documentation has not been released. However, a published paper is extensive enough for review. Schock's model[41] is a rigorous third-order design method. He applied the same differential equations as Urieli, but his method of computer modeling was different. Schock employed a finite-difference, explicit-forward integration technique. He further developed a special scheme for enhanced mathematical stability and for accelerated convergence to a steady state cycle. However, no details were given in the paper. Furthermore, the option of an inertialess gas was provided for a faster but less accurate calculation.

The model has been applied to a free-piston Stirling engine built by Sunpower, Inc., for DOE. The model predicted (1) no significant differences in cyclic pressure variation between expansion and compression spaces; (2) nonuniform mass flow rate, which showed gas streaming out of both ends of the regenerator at certain parts of the cycle; and (3) highly fluctuating gas temperature profiles in each space. In addition, the model is capable of calculating cyclic heat flow profiles, mechanical power outputs, and energy balances in tabulated forms or three-dimensional plots.

3.2.9 Model by Gedeon (1978) - rigorous analysis

In a highly descriptive and informative paper on the optimization of Stirling cycle machines, Gedeon[42] of Sunpower, Inc., describes an optimization computer program used for the development and performance evaluation of Sunpower's 1-kW engine (SP1KE). The computer program consists of two parts: an optimization scheme and a third-order thermodynamic simulation. It is the sole interest of this review to isolate and concentrate on the thermodynamic analysis. Gedeon's third-order analysis differs from the third-order methods used by most other investigators in two important aspects. First, the working gas is divided into only six nodes: two in the regenerator and one each in the heater, cooler, expansion space, and compression space. The computed results changed by only a few percent when additional control volumes were used in the analysis. Thus, it was decided that the model with six control volumes for the working gas was sufficiently accurate for their optimization study. In addition, a nodal network was included to account for energy paths in the piston, displacer, and cylinder walls. Losses caused by shuttle heat transfer, gas spring hysteresis, and wall conduction could all be accounted for.

The other major difference in Gedeon's thermodynamic simulation was his integration technique. Gedeon used an implicit numerical method to integrate the continuity, momentum, and energy equations. In implicit integrations, the dependent nodal variables at a new time are determined simultaneously. This requires the inversion of a large matrix at each time step. However, long time steps can be used because implicit integration techniques are always numerically stable. Gedeon's numerical method required about 200 time steps per piston revolution to maintain an integration accuracy of three significant figures. Losses due to gas leaks past piston and displacer seals were properly accounted for in his mass and energy balances. Similar to other investigators, Gedeon included a special subroutine to accelerate the convergence of the integration toward a cyclic steady state solution. At the end of each piston cycle, the nodal temperatures were adjusted based on average temperatures and net energy accumulation during the cycle. A steady solution was found in about ten piston cycles.

Hardware development at Sunpower, Inc., has complemented and augmented the development of their computer models. Gedeon claims that the agreement between their experimental, free-piston Stirling engines and computer predictions is within ±10% for all measurable parameters. No further details were provided by Gedeon.

3.2.10 Model by Zacharias (1977)

This review was made possible through an ORNL translation of a paper written in German by F. Zacharias[43] of MAN/MWM. A full evaluation of the model was not possible because few details about the model were given in the paper. The model described by Zacharias utilizes a third-order design method. However, it was not stated whether a complete set of conservation equations (continuity, momentum, energy) was solved or if the gas inertia or kinetic energy terms were neglected in the equations. Like other third-order models, the engine was partitioned into a network of control volumes. The state of the gas in each control volume was defined by three variables: pressure, temperature, and mass flow rate. After the conservation equations were converted from differential to finite-difference equations, they were integrated by using an explicit numerical technique. A special algorithm was needed to accelerate the convergence of the regenerator nodal temperatures toward a cyclic steady state solution; but no details were provided.

Zacharias applied the model to a four-cylinder, double-acting Stirling engine. The predicted gas temperatures and pressures were presented on three-dimensional plots as functions of position and time. A comparison between model predictions and experimental measurements was not included in the paper.

3.3 Method of Characteristics

Two models were reviewed in this category. Both models are based on some simplifying approximations that decouple one of the conservation equations from the solution of the other two. The first model solves simultaneously the conservation equations of mass and momentum. The second model is based on the simultaneous solution of the conservation equations of mass and energy.

3.3.1 Model by Organ (1981)

Organ[7,8] of Cambridge University modeled an alpha-configuration Stirling engine as a series of ducts that can branch out into parallel paths and can have gradually changing flow areas. The gas velocity and pressure were assumed to be functions of axial position and time. The temperature of the gas, however, was assumed to depend only on position (i.e., the gas at a particular location is isothermal). The gas temperature distribution was specified a priori and was set equal to the heater wall temperature in the expansion cylinder and heater, the cooler wall temperature in the compression cylinder and cooler, and a straight-line transition from hot-end to cold-end temperature in the regenerator. For these assumptions, the flow is defined by the conservation equations of mass and momentum. When these equations are solved by the method of characteristics, the characteristic curves are found to be the Mach lines. In the physical plane (position-time), there are two families of Mach lines that propagate either rightward or leftward at the local acoustic velocity relative to a moving fluid particle. The conservation equations are integrated numerically along the Mach lines. This technique ensures that pressure information propagates through the gas at the speed of sound.

Organ[7] applied his computer model to a Stirling machine that has rather long heat exchangers and uses air as the working fluid. He presented a plot of the Mach line net for an instantaneous startup to 4000 rpm. Some features of the solution pertaining to the initial half revolution of crankshaft motion include: (1) a fan of rarefaction waves from the impulsive withdrawal of the compression-space piston, (2) a triangular dead region where the gas remains undisturbed for a finite period of time after the instantaneous startup, and (3) a substantial change in the gradient of the Mach lines across the regenerator caused by the temperature gradient over that component. For the conditions in Organ's example, it took about 65° of crankshaft rotation for the pressure information to travel from one piston to the other. However, this angle would have been considerably smaller if the heat exchangers were shorter, the engine speed was lower, or the acoustic velocity of the working fluid was higher (using helium or hydrogen rather than air).

Organ also presented a plot of predicted work output per cycle vs engine speed. The net output was computed by integrating the pressures at the surfaces of the compression and expansion pistons (after a cyclic steady state solution was reached) with respect to the piston positions. When Organ compared his predictions with the work output per cycle computed from the ideal Schmidt analysis, it was evident that the effects of fluid friction and inertia are very important, especially at high engine speeds.

3.3.2 Model by Larson (1981)

Larson[44] of Cleveland State University has developed a Characteristic Dynamic Energy Equations (CDEE) computer model based on the method of characteristics. Larson formulated his analysis in terms of three variables: gas density, velocity, and temperature. The analysis was simplified by decoupling the momentum equation from the continuity and energy equations and neglecting kinetic energy in the energy equation. Approximate expressions for the gas velocity and its spatial derivative were derived by assuming a spatially uniform density and then correcting for the effects of pressure drop. The approximate velocity expression enabled Larson to (1) separate the momentum equation from the system of simultaneous equations and (2) compute pressure directly from the momentum equation. The continuity and energy equations along with the total differentials of density and temperature form a system of hyperbolic partial differential equations. The two characteristic curves for this system of equations are the gas velocity and the gas velocity multiplied by the heat capacity ratio. The characteristic directions were used to transform the partial differential equations into a set of ordinary differential equations that are valid along the characteristic curves. These equations were solved numerically using a fourth-fifth order Runge-Kutta integration technique.

Larson[45] applied his model to the GPU-3 configuration. Gas temperatures were computer plotted as a function of position and crank angle. Pressure-volume diagrams for the compression and expansion cylinders were also presented for a typical set of operating conditions. When compared with GPU-3 experimental measurements, the CDEE model overpredicted power output by no more than 10% over the entire frequency range from 1000 to 3500 rpm. However, no details were provided about the heat transfer and fluid friction correlations used in the model to achieve this good agreement.

4. SUMMARY

In this state-of-the-art review, four distinct Stirling engine design methods were identified based on the degree of sophistication: approximate (first-order), decoupled (second-order), nodal (third-order), and method of characteristics. First-order methods are good for preliminary system analysis. Second-order methods are good for interactive design and optimization. Third-order methods are good for detailed simulation of the mass, pressure, and temperature distributions in a Stirling engine. The method of characteristics first determines the characteristic curves of the conservation equations and then integrates the equations along the characteristic curves. This method can account for the finite velocity of pressure waves.

All of the engine design methods, except for first-order analysis, were reviewed. The number of models reviewed (19 total) is approximately twice the number reviewed previously by others (10 total). Among the 19 models, 7 are second-order, 10 are third-order, and 2 use the method of characteristics. Based on a carefully designed model review format, fundamental assumptions, limitations, and applicability of the individual computer models were discussed.

For quick cross-reference, Tables 1 and 2 summarize the attributes of the second- and third-order models. The models are compared in terms of classification, simplification, code listing availability, and model validation. Classification of the second-order models reveals that adiabatic analysis overwhelms the other second-order classifications. Five of the second-order models use adiabatic analysis, one uses isothermal analysis, and one uses semi-adiabatic analysis. For the third-order models, approximate analyses (six) outnumber rigorous analyses (three) by a two-to-one margin.

Table 1. Summary of Stirling engine mathematical models - second-order design methods

Principal investigator	Present affiliation	Second-order classification	Code listing availability	Model validation	References
W. R. Martini	Martini Engineering	Isothermal	Yes[a] (Ref. 3)	Yes, GM GPU-3 and 4L23 engines (Ref. 3)	1, 2, 3, 4
E. B. Qvale	Laboratory for Energetics, Denmark; original work done at MIT	Adiabatic	Not published	Yes, Allison PD-67A engine (Ref. 19)	19, 20
P. A. Rios	General Electric Company; original work done at MIT	Adiabatic	Yes, as modified by Martini (Ref. 3)	Yes, GM 4L23 engine, by Martini (Ref. 3)	21
K. Lee	Foster-Miller Associates	Adiabatic	Not published	Yes, GM GPU-3 engine (Ref. 22)	22, 23
R. Shoureshi	Wayne State University	Adiabatic	Yes (Ref. 24)	Yes, GM GPU-3, Allison PD-67A, and Philips engines (Ref. 24)	24, 25
T. J. Heames	Argonne National Laboratory	Adiabatic	Yes,[b] from National Energy Software Center	Yes, GM GPU-3 engine (Ref. 26)	26
B. Feurer	Unknown; original work done at MAN/MWM, West Germany	Semi-adiabatic	Not published	Not published	27

[a]Computer code (on a floppy disk) acquired by ORNL.

[b]Computer code (on tape) acquired by ORNL.

Table 2. Summary of Stirling engine mathematical models - third-order design methods

Principal investigator	Present affiliation	Numerical simplifications	Code listing availability	Model validation	References
T. Finkelstein	TCA Stirling Engine R&D Company	No gas inertia or kinetic energy	Available for use through CDC Cybernet computer	Yes, but not published	28
R. C. Tew	NASA-Lewis Research Center	Common pressure, no kinetic energy	Yes[a] (Refs. 2 and 29)	Yes, GM GPU-3 engine (Ref. 31), USS P-40 engine (Ref. 32)	29, 30, 31, 32
J. E. Giansante	Mechanical Technology, Inc.	Common pressure, no kinetic energy	Yes (Ref. 33)	Limited, DOE 1-kW free-piston engine (Ref. 33)	33
W. S. Chiu	General Electric Company	No gas inertia or kinetic energy	Not published	Yes, GE Proto 1 and 2 free-piston engines (Ref. 34)	34
A. Azetzu	University of Tokyo, Japan	Common pressure, no kinetic energy	Not published	Yes, University of Tokyo test engine (Ref. 35)	35
T. G. Vanderbrug	Unknown, original work done at Jet Propulsion Laboratory	No gas inertia	Yes (Ref. 36)	Limited, JPL Research Engine (Ref. 37)	36, 37
I. Urieli	Sunpower, Inc.; original work done at University of Witwatersrand, S. Africa	Rigorous	Yes (Refs. 2 and 39)	Yes, University of Witwatersrand test engine (Ref. 40)	38, 39, 40
A. Schock	Fairchild Industries	Rigorous	Not published	Yes, but not published	41
D. R. Gedeon	Sunpower, Inc.	Rigorous	Not published	Yes, Sunpower free-piston engines, not published	42
F. Zacharias	Unknown; original work done at MAN/MWM, West Germany	Unknown	Not published	Not published	43

[a]A copy of early NASA-Lewis computer program acquired by ORNL.

5. CONCLUSIONS WITH RECOMMENDATIONS

This section provides some broad perspectives and recommendations about analyses of Stirling engines. In the course of our study, the following were established: (1) a state-of-the-art review of Stirling engine thermodynamic models, (2) an information center, and (3) a qualitative comparison between the numerous models.

A full and complete assessment was not attempted in this report because the limited time available did not allow us to obtain all of the documentation (especially some of the more obscure items such as theses, reports, and lecture notes) relating to some codes. In addition, difficulties were encountered with incomplete draft reports. Thus, it is recommended that comprehensive literature surveys should be continued, domestically and abroad as well. For instance, on the domestic front, there are numerous companies that are or have been outstanding in Stirling engine design, application, and manufacturing; yet their documentation is not available. Typical companies are Sunpower, Inc., and Fairchild Industries. For those companies abroad, the situation is worse. Information from many active and leading companies in Stirling engine research and development is scarce or not even released because of proprietary or security restrictions. Examples include Philips, Harwell, MAN/MWM, the French, and the Japanese. Extensive communication, information exchange, and program cooperation may improve this problem, but as long as these organizations see a commercial or military future for Stirling machines, some proprietary restrictions are likely to exist.

Utilization of a detailed design method does not ensure enhanced model performance. According to our review, there is at present no evidence to claim that the existing third-order and method of characteristics analyses are superior to the second-order methods. However, it should be pointed out that many of the models required arbitrary corrections for the friction factor and/or heat transfer correlations to make the power output and efficiency predictions fit the validation data better. Whether these correction factors represent weaknesses in the models themselves or weaknesses in the friction and heat transfer correlations is a question that must be resolved. After this question is answered, the rigorous thermodynamic models will be more accurate and more generally applicable.

Contradictory opinions exist among the model developers about differences between the integration techniques used in the nodal analyses and the method of characteristics. The point in question is the speed at which thermodynamic information (pressure, temperature, etc.) propagates through the engine. As discussed earlier, the nodal integrations use a system of fixed grids and uniform time steps and can be classified into two major categories: explicit and implicit techniques. In explicit (forward-differencing) techniques, thermodynamic information propagates only from one node to an adjacent node during each time step. The node spacing and time step, therefore, determine the speed at which information propagates through the grid system. The explicit techniques are sometimes plagued by numerical instabilities, if the time steps are improperly chosen. In contrast, the implicit (backward-differencing) technique is always numerically stable, regardless of grid spacings and time steps. The implicit technique solves the finite difference equations describing the state of the working fluid by calculating the condition of the gas in each cell at a particular time from the condition of the gas in all other cells at that time. Consequently, thermodynamic information can propagate from one end of the engine to the other during one time step.

Numerical integrations using the method of characteristics are normally based on fixed time steps and floating grids. The choice of grid spacing depends on the characteristic curves of the governing equations. In Organ's model, in which the method of characteristics was applied to the conservation equations of mass and momentum, the nodes are spaced so that the pressure information propagates at the local acoustic velocity relative to the local gas velocity.

It may be very important to account for the fact that pressure waves propagate at the speed of sound when predicting the performance of certain engines, especially ones that have long heat exchangers and operate at high frequencies. However, this effect may not be very important in other engines, and the nodal integration techniques may provide sufficient accuracy. This is a dilemma that will only be resolved by further investigation.

We are not in a position now to rank with confidence the models reviewed. The only fair way to compare the models would be to run all of the codes on the same computer and compare their predictions with data from well-defined experimental Stirling engines. Some work along these lines was begun by ANL.[46] However, this is a difficult task for three apparent reasons. First, every model has its unique attributes and may also be applicable only to a limited number of engine configurations. Second, acquisition of codes would have to be limited to nonproprietary codes. Finally, a lack of well-defined experimental data is a hindrance. Numerous investigators have published experimental data, but many of them have not provided enough information about engine dimensions, parameter definitions, and even the operating conditions to allow a meaningful comparison between simulation and experiment.

It is obvious that a large variety of thermodynamic models for Stirling engine analysis has been developed. They range in complexity from the simple first-order models up through the rigorous third-order and method of characteristics models. It does not seem necessary to develop any new thermodynamic models. Time and effort would be better spent by increasing our understanding of the existing models. One final observation is that validation of the thermodynamic models has been limited mainly to kinematic engines. When experimental data from free-piston Stirling engines have been used to validate the models, the experimentally determined dynamic parameters (such as frequency, phase angle, and piston amplitudes), rather than predicted values from a separate free-piston dynamics model, have usually been used as inputs to the thermodynamic models. Analyses of free-piston dynamics have been explored to a much lesser extent than those of Stirling engine thermodynamics, but it is an area that is important and needs additional studies.

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W. S. Chiu - General Electric Company

D. R. Gedeon - Sunpower, Inc.

F. W. Hoehn - Rockwell International (former colleague of T. G. Vanderbrug)

V. H. Larson - Cleveland State University

W. R. Martini - Martini Engineering

A. J. Organ - Cambridge University

E. B. Qvale - The Technical University of Denmark

P. A. Rios - General Electric Company

R. Shoureshi - Wayne State University

R. C. Tew - NASA-Lewis Research Center

REFERENCES

1. W. R. Martini, *Stirling Engine Design Manual*, NASA-CR-1353 82, Natl. Aeronautics and Space Administration, April 1978.

2. W. R. Martini, *Thermodynamic Design of Stirling Engines by Computer*, Martini Engineering, Richland, Washington, 1980.

3. W. R. Martini, *Stirling Engine Design Manual – Second Edition*, to be published by NASA-Lewis.

4. W. R. Martini, "Validation of Published Stirling Engine Design Methods Using Engine Characteristics from the Literature," pp. 2245–50 in *Proceedings of the 15th IECEC*, Paper No. 809449, 1980.

5. I. Urieli, "A Review of Stirling Cycle Machine Analysis," pp. 1086–90 in *Proceedings of the 14th IECEC*, Paper No. 799236, 1979.

6. G. Walker, *Stirling Engines*, Clarendon Press, Oxford, 1980.

7. A. J. Organ, "Gas Dynamics in the Temperature-Determined Stirling Cycle," *J. Mech. Eng. Sci.* 23(4), 207–16 (August 1981).

8. A. J. Organ, "Gas Dynamics of Stirling Cycle Machines, Stirling Engines – Progress Toward Reality," pp. 131–40 in *Mechanical Engineering Publications Limited*, London (March 1982).

9. J. R. Senft, "A Simple Derivation of the Generalized Beale Number," pp. 1652–55 in *Proceedings of the 17th IECEC*, Paper No. 829273, 1982.

10. T. Finkelstein, *Generalized Thermodynamic Analysis of Stirling Engines*, SAE Paper No. 118B, Society of Automotive Engineers, January 1960.

11. A. H. Shapiro, *The Dynamics and Thermodynamics of Compressible Fluid Flow - Vols. I and II*, Ronald Press, New York, 1953.

12. H. W. Liepmann and A. Roshko, *Elements of Gasdynamics*, John Wiley and Sons, New York, 1957.

13. J. S. Rauch, "Harmonic Analysis of Stirling Engine Thermodynamics," pp. 1696–1700 in *Proceedings of the 15th IECEC*, Paper No. 809335, 1980.

14. R. Berggren, *Analysis Documentation – STENSY, Interim No. 1*, MTI Report No. 82 ASE286ER46, Mechanical Technology, Inc. (March 1983).

15. N. E. Andersen, *Stirling Engine Modular Analysis Program (SEMAP)*, Laboratory for Energetics, Technical University of Denmark, RE 79-9, 1979.

16. E. J. Sirett, "The Gas Dynamics of the Stirling Machine," first year report on postgraduate work, Cambridge University Engineering Dept., 1981.

17. R. Vincent, W. Rifkin, and G. Benson, "Analysis and Design of Free-Piston Stirling Engines — Thermodynamics and Dynamics," pp. 1686–95 in *Proceedings of the 15th IECEC*, Paper No. 809334, 1980.

18. R. Howlett, *A Digital Computer Simulation of the Thermomechanical Generators*, AERE-M2294, Atomic Energy Research Establishment, United Kingdom, 1970.

19. E. B. Qvale and J. L. Smith, Jr., "A Mathematical Model for Steady Operation of Stirling-Type Engines," *J. Eng. Power*, 45–50 (January 1968).

20. E. B. Qvale, *An Analytical Model of Stirling-Type Engines*, Ph.D. Thesis, Massachusetts Institute of Technology, 1967.

21. P. A. Rios, *An Analytical and Experimental Investigation of the Stirling Cycle*, Ph.D. Thesis, Massachusetts Institute of Technology, 1969.

22. K. Lee, I. P. Krepchin, and W. M. Toscano, "Thermodynamic Description of an Adiabatic Second Order Analysis for Stirling Engines," pp. 1919–24 in *Proceedings of the 16th IECEC*, Paper No. 819794, 1981.

23. K. Lee et al., "Performance Loss Due to Transient Heat Transfer in the Cylinders of Stirling Engines," pp. 1706–1909 in *Proceedings of the 15th IECEC*, Paper No. 809338, 1980.

24. R. Shoureshi, *Analysis and Design of Stirling Engines for Waste-Heat Recovery*, Ph.D. Thesis, Massachusetts Institute of Technology, 1981.

25. R. Shoureshi, "Simple Models for Analysis and Design of Practical Stirling Engines," pp. 1647–51 in *Proceedings of the 17th IECEC*, Paper No. 829272, 1982.

26. T. J. Heames et al., "A User Oriented Design System for Stirling Cycle Codes," pp. 1681–87 in *Proceedings of the 17th IECEC*, Paper No. 829278, 1982.

27. B. Feurer, "Degrees of Freedom in the Layout of Stirling Engines," presented at Von Kaman Institute for Fluid Dynamics — Lecture Series 53, Feb. 12–16, 1973.

28. T. Finkelstein, "Computer Analysis of Stirling Engines," pp. 933–41 in *Proceedings of the 10th IECEC*, Paper No. 759140, 1975.

29. R. Tew, K. Jefferies, and D. Miao, *A Stirling Engine Computer Model For Performance Calculations*, NASA TM-78884, Natl. Aeronautics and Space Administration, July 1978.

30. R. C. Tew, *Computer Program for Stirling Engine Performance Calculations*, NASA/TM-82960, Natl. Aeronautics and Space Administration, January 1983.

31. R. C. Tew, L. G. Thieme, and D. Miao, *Initial Comparison of Single Cylinder Stirling Engine Computer Model Predictions with Test Results*, NASA TM-79044, Natl. Aeronautics and Space Administration, March 1979.

32. W. A. Tomazic, *Supporting Research and Technology of Automotive Stirling Engine Development*, NASA TM-81495, Natl. Aeronautics and Space Administration, April 1980.

33. J. E. Giansante, *A Free Piston Stirling Engine Performance Code 81TR17*, prepared for NASA-Lewis by Mechanical Technology, Inc., November 1980.

34. General Electric, *The Thermodynamic Program (TDP) for Free-Piston Stirling Engine/Linear Compressor Simulation*, GE No. GFHP-82-048, 1982.

35. A. Azetsu, N. Nakajima, and M. Hirata, "Computer Simulation Model for Stirling Engine," pp. 57–63 in *Stirling Engines – Progress Towards Reality*, Mechanical Engineering Publications Limited, London, March 1982.

36. J. G. Finegold and T. G. Vanderbrug, *Stirling Engines for Undersea Vehicles – Final Report*, JPL No. 5030-63, Jet Propulsion Laboratory, March 1977.

37. F. W. Hoehn, B. D. Nguyen, and D. D. Schmit, "Preliminary Test Results with a Stirling Laboratory Research Engine," pp. 1075–81 in *Proceedings of the 14th IECEC*, Paper No. 799233, 1979.

38. I. Urieli, C. J. Rallis, and D. M. Berchowitz, "Computer Simulation of Stirling Cycle Machines," pp. 1512–21 in *Proceedings of the 12th IECEC*, Paper No. 779252, 1977.

39. I. Urieli, *A Computer Simulation of Stirling Cycle Machines*, Ph.D. Thesis, University of Witwatersrand, South Africa, 1977.

40. D. M. Berchowitz and C. J. Rallis, "A Computer and Experimental Simulation of Stirling Cycle Machines," pp. 1730–38 in *Proceedings of the 13th IECEC*, Paper No. 789111, 1978.

41. A. Schock, "Nodal Analysis of Stirling Cycle Devices," pp. 1771–79 in *Proceedings of the 13th IECEC*, Paper No. 789191, 1978.

42. D. R. Gedeon, "The Optimization of Stirling Cycle Machines," pp. 1784–90 in *Proceedings of the 13th IECEC*, Paper No. 789193, 1978.

43. F. Zacharias, "Further Stirling Engine Development Work – Part 1," *Motortech. Z.* 38(9), 371–77 (1977).

44. V. H. Larson, "Characteristic Dynamic Energy Equations for Stirling Cycle Analysis," pp. 1942–47 in *Proceedings of the 16th IECEC*, Paper No. 819798, 1981.

45. V. H. Larson, "Computation Techniques and Computer Programs to Analyze Stirling Engines Using Characteristic Dynamic Energy Equations," pp. 1710–15 in *Proceedings of the 17th IECEC*, Paper No. 829283, 1982.

46. J. E. Ash and T. J. Heames, "Comparative Analysis of Computer Codes for Stirling Engine Cycles," pp. 1936–41 in *Proceedings of the 16th IECEC*, Paper No. 819797, 1981.

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