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Z-Source Inverter for Fuel Cell Vehicles

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EXECUTIVE SUMMARY

The objective of this project is to develop and demonstrate a low-cost, efficient, and reliable inverter for traction drives of fuel cell vehicles (FCVs). Because of the wide voltage range of the fuel cell, the inverter and the motor need to be oversized to accommodate the great constant power speed ratio. The Z-source inverter could be a cheap and reliable solution for this application. This report summarizes the results and findings during the development of the Z-source inverter.

Currently, two types of inverters are used in FCV and hybrid electric vehicle (HEV) traction drives: the traditional pulse width modulation (PWM) inverter and the dc/dc–boosted PWM inverter. For FCVs, the fuel cell voltage to the inverter decreases with an increase in power drawn from the fuel cell. Therefore, the obtainable output voltage of the traditional PWM inverter is low at high power for this application, so an oversized inverter and motor must be used to meet the requirement of high-speed, high-power operation. The dc/dc–boosted PWM inverter does not have this problem; however, the extra dc/dc stage increases the complexity of the circuit and the cost and reduces the system efficiency. To demonstrate the superiority of the Z-source inverter for FCVs, a comprehensive comparison of the three inverters was conducted. It shows that the Z-source inverter provides higher efficiency and lower cost.

The detailed design process of the Z-source inverter is provided. The design includes the inductor design, capacitor selection, device selection, and 3-dimensional design. A dc rail clamp circuit was implemented to reduce the voltage overshoot across the power device. The designs of the dc rail clamp circuit and the gate drive board, as well as of the sensor boards, are all provided.

Several PWM schemes with shoot-through are proposed and compared. The PWM scheme with the maximum constant boost, different from other PWM schemes proposed, results in less switching loss. A 30-kW prototype was built, and primary test results are provided. The test results demonstrated that the original objective of the project had been achieved: high efficiency (greater than 97%), low cost (with minimal device ratings), improved reliability (because no dead time is needed), and wide constant power speed ratio (1.55 times that of the traditional PWM inverter, thanks to the voltage boost).

It was discovered during the testing that the Z-source inverter without a battery has self-boost functioning when the traction motor is operating at low speed, low power, and low power factor. This self-boost function was initially deemed a problem; however, it actually is a needed function...
for successful fuel cell startup, especially during a freeze start. In addition, this self-boost can be totally controllable when a battery is incorporated into the inverter system.

In summary, the results of the project demonstrate the many unique features of the Z-source inverter and its high feasibility for use in FCVs. As the automotive industry has been urged to develop HEVs based on a combination of internal combustion engine and batteries as a bridging technology to FCVs, issues and possibilities are addressed for the use of the Z-source inverter in ICE HEV traction drive systems.
1. Introduction

1.1 Background

The fuel cell, a clean energy source, provides much higher efficiency than the traditional internal combustion engine (ICE), which potentially makes the fuel cell electric drive system the next-generation traction system. Fuel cells have a unique polarization curve, as shown in Fig. 1.1. The output voltage of the fuel cell declines dramatically when the output current increases. The output voltage of the fuel cell at the maximum power point is about half of the open load voltage. Both permanent magnet and induction machines for fuel cell vehicle (FCV) traction drives require high voltage at high speed and high power. Thus to achieve high speed and high power, the inverter and the motor must be oversized if only a traditional pulse width modulation (PWM) inverter is used as the power converter.

![TYPICAL FUEL CELL POLARIZATION CURVE](image)

Fig. 1.1. Typical fuel cell polarization curve.

In addition, the fuel cell has a relatively slow response and unidirectional power flow. Therefore, an energy storage device is always needed to handle load dynamics and regenerative braking. During cold (or freeze) start, the fuel cell has to be operated at high current and low voltage to heat up the fuel cell stack. After being fully started, the fuel cell prefers constant fixed-power operation for balance-of-plant purposes.

Figures 1.2 (a) and (b) show the two existing FCV traction drive system configurations with battery or ultra-capacitor [1–6]. At a low speed and low power, the fuel cell is turned off, and the
vehicle is normally operated and powered from the battery when the battery is fully charged. When the battery is low or during medium- and high-power operation, the fuel cell is started and operates at its preferred constant power for maximum efficiency and easy balance of plant. Figures 1.3 (a), (b), (c), and (d) show the typical operating modes of an FCV [3–7].

Fig. 1.2. Existing traction drive system configurations of fuel cell vehicles.
Accordingly, the Z-source inverter [8] has three configurations for FCVs with battery or ultra-capacitor [9]. Figure 1.4 (a) shows the configuration using a high-voltage battery or ultra-capacitor as a substitute for one of the capacitors in the Z-source. Figures 1.4 (b) and (c) show the configurations with a low-voltage battery and using a dc/dc converter.
Fig. 1.4. Traction drive system configurations of the Z-source inverter for fuel cell vehicles with battery.

The present prototype, as shown in Fig. 1.5, has no energy storage device, battery or ultra-capacitor and thus cannot handle load dynamics and regeneration. The purpose of this prototype and its testing is to confirm the Z-source inverter’s features, functions, and performance, such as voltage boost, efficiency, and low harmonics.
1.2 Overview of FCVs and Fuel Cell Balance of Plant

As mentioned, FCV control/operation modes and the fuel cell stack’s balance of plant are very complicated to maximize the system efficiency and to enable the fuel cell stack to start quickly and reliably and operate safely and efficiently. A set of slides from Dr. Fred Flett, vice-president for engineering of Ballard Corporation (Figs. 1.6 through 1.12), summarize the challenges and provide an overview of FCVs and the fuel cell stack balance of plant. The content of these slides can be summarized in the following points: (1) an FCV needs a battery to handle transients; (2) the fuel cell stack must be operated in a constant power region to maximize efficiency and facilitate balance of plant; (3) the fuel cell must be shut off at low speed and low power and when the battery is fully charged; and (4) frequent startup of the fuel cell presents a huge challenge to power electronics, especially freeze starts, because it requires that the fuel cell be operated at low voltage, high current for successful startup. This is almost impossible for the traditional PWM inverter. The voltage boost function is particularly preferable during freeze start.

The original objective of this project is to develop a Z-source inverter and demonstrate its cost, efficiency, and reliability features for FCV traction drives. During the testing of the Z-source inverter prototype, it was discovered that the Z-source inverter without a battery has self-boost functioning when the traction motor is operating at low speed, low power, and low power factor. This self-boost function was initially deemed a problem; however, it turned out to be a needed function for successful fuel cell startup, especially during freeze start.
Fig. 1.6. Toyota FCV system configuration.

Fig. 1.7. Vehicle system architectures. The “lean” system is not commonly used because of low efficiency and limited regen capability. Most FCVs are “FC following hybrid” systems.
Fig. 1.8. Polarization curve showing voltage collapse when falling below the minimum system voltage and exceeding the maximum current density.

Fig. 1.9. Operating point determines useful power and waste heat. The fuel cell has to be operated to balance the plant and produce the maximum power possible simultaneously.
Fig. 1.10. Thermal and power density constraints showing the fuel cell has to be operated in a certain region of the polarization curve. For example, the fuel cell cannot be operated at no-load (or open circuit) voltage because of thermal constraints during normal operation and auxiliary power consumed by the fuel cell itself.

Fig. 1.11. Freeze (cold) start requires low fuel cell voltage and high current. The lower the acceptable cell voltage during freeze startups, the more waste heat is produced per water molecule and the more likely we are to be successful. The traditional PWM inverter cannot draw enough current at low dc input voltage even with a modulation index of 1.0, an additional boost converter is required.
Fig. 1.12. A decreased minimum system voltage operation can generate more waste heat, thus providing fast, reliable freeze starts. However, freeze starts require more functionality in power electronics, that is, large voltage boost for freeze starts. Therefore, a voltage boost converter is highly preferred after the fuel cell if the traditional PWM inverter is used for a traction drive.

1.3 Inverter Specifications

Specifications of the inverter for fuel cell FCVs are as follows:

1. Continuous power: 30 kW
2. Peak power: 55 kW for 18 seconds
3. Inverter efficiency >97% at 30 kW
4. Input fuel cell voltage: 0–420 V dc

1.4 Layout of the Report

This report documents the development and findings of the project.

In Section 2, a comprehensive comparison of the Z-source inverter and traditional inverters is conducted. Switching device power, efficiency, passive component requirements, and constant power speed ratio (CPSR) are used as benchmarks for the comparison.
The detailed design of the inverter is described in Section 3. To minimize the switching loss, a new shoot-through PWM control is presented in Section 4.

The testing results of the inverter are provided in Section 5.

Because the Z-source inverter prototype has no battery, a self-boost phenomenon was observed when the modulation index or power factor was low during the test. This self-boost phenomenon is analyzed theoretically and discussed in Section 6. Self-boost is not a problem for fuel cell-battery systems.
2. Comparison of Z-Source and Traditional Inverters for FCVs

2.1 Introduction

Currently, two existing inverter topologies are used for HEVs and FCVs: the conventional 3-phase PWM inverter and a 3-phase PWM inverter with a dc/dc boost front end. Because of a wide voltage change and the limited voltage level of the battery and/or fuel cell stack, the conventional PWM inverter topology imposes high stresses on the switching devices and motor and limits the motor’s CPSR. The dc/dc–boosted PWM inverter topology can alleviate the stresses and limitations; however, it has other problems, such as the high cost and complexity associated with the two-stage power conversion. This present project is to investigate and develop a new inverter topology, the Z-source inverter for FCVs. This section discusses a comprehensive comparison of the Z-source inverter versus the two existing inverter topologies, performed using a 50-kW (max) fuel cell stack as the prime energy source and a 34-kW Solectria AC55 induction motor as the traction drive motor. The comparison results show that the Z-source inverter can increase conversion efficiency by 1% over a wide load range, extend CPSR by 1.55 times, and minimize the switching device power rating (SDPR), a cost indicator, by 15%. Simulation models and results will be reported to verify the comparison.

2.2 System Specifications

The traction drive system is powered by a fuel cell stack with the characteristic curve shown in Fig. 2.1. The traction motor is a Solectria AC55 (Fig. 2.2) induction machine.
The maximum output power of the fuel cell stack is 50 kW when its output voltage is 250 V and output current is 200 A. The maximum output voltage (or open-circuit voltage) of the fuel cell is 420 V when it outputs no current. Therefore, Fig. 2.1 can be viewed as a normalized V-I curve with 420 V as the base voltage value and 200 A as the base current value. The specification of the motor is as follows:

- Peak torque: 240 N-m
- Maximum current: 250 A rms
- Continuous torque: 55 N-m
- Continuous power: 34 KW
- Peak efficiency: 93%
- Peak electrical p8 kW at voltage of 312 Vdc
- Nominal speed: 2.5 K rpm
- Maximum speed: 8.0 K rpm

The desired nominal output power is 30 kW, and the maximum power is 50 kW.

2.3 System Configurations for Comparisons

As previously mentioned, three different inverter system configurations are to be investigated: the conventional PWM inverter, dc/dc boost plus PWM inverter, and the Z-source inverter. Their system configurations are shown in Figs. 2.3 (a), (b), and (c), respectively.

2.4 Comparison Items, Conditions, Equations, and Results

2.4.1 Total Switching Device Power Rating Comparison

In an inverter system, each switching device has to be selected according to the maximum voltage impressed and the peak and average current going through it. To quantify the voltage and current stress (or requirement) of an inverter system, SDPR is introduced. The SDPR of a switching device/cell is expressed as the product of voltage stress and current stress. The total SDPR of an inverter system is defined as the aggregate of the SDPRs of all the switching devices used in the circuit. Total SDPR is a measure of the total semiconductor device requirement and is thus an important cost indicator for an inverter system. The definitions are summarized as follows:
Total average SDPR = \((SDPR)_{av}\) = \(\sum_{i=1}^{N} V_i I_{i\_average}\), and

Total peak SDPR = \((SDPR)_{pk}\) = \(\sum_{i=1}^{N} V_i I_{i\_peak}\), where \(N\) is the number of devices used.

The average and peak SDPRs of the conventional PWM inverter are, respectively (see the appendices for detailed derivations),

\[
(SDPR)_{av} = \frac{8V_{\text{max}} P_o}{\cos \phi V_i \pi M} \quad \text{and} \quad \text{(2.1)}
\]
\[(SDPR)_{pk} = \frac{8V_{\text{max}}P_o}{\cos \varphi V_i M} .\]  \hspace{1cm} (2.2)

The average and peak SDPR of the dc/dc boost plus PWM inverter are

\[(SDPR)_{av} = \frac{8P_o}{\cos \varphi \varphi M} + \frac{P_o}{V_i} \cdot V_{DC} \] \hspace{1cm} (2.3)

\[(SDPR)_{pk} = \frac{8P_o}{\cos \varphi M} + \frac{P_o}{V_i} \cdot V_{DC} .\] \hspace{1cm} (2.4)

The average and peak SDPR of the Z source inverter are

\[(SDPR)_{av} = \frac{2P_o(2 - \sqrt{3}M)}{(\sqrt{3}M - 1)} + \frac{4\sqrt{3}P_o}{\cos \varphi \pi} \] \hspace{1cm} (2.5)

\[(SDPR)_{pk} = \frac{4P_o}{\sqrt{3}M - 1} + \frac{4P_o}{\cos \varphi M} .\] \hspace{1cm} (2.6)

In these equations, \(P_o\) is the maximum output power; \(V_{\text{max}}\) is the maximum output voltage of the fuel cell stack; \(\cos \varphi\) is the power factor of the motor at maximum power; \(V_i\) is the fuel cell stack output voltage at maximum power; \(M\) is the modulation index; \(V_{\text{dc}}\) is the output voltage of the boost converter in the dc/dc–boosted inverter, which is greater than \(V_{\text{max}}\).

Based on these equations, a comparison of total SDPRs with the following specifications is performed and summarized in Table 2.1:

- Maximum power: 50 kW
- Motor power factor at maximum power: 0.9
- Output voltage of the boost converter: 420 V
- Modulation index of conventional PWM inverter and dc/dc boost + PWM inverter: 1 at maximum power
- Modulation index of Z source inverter: 0.92 [10] (to keep the voltage stress of the switches lower than 420 V)
Table 2.1. Switching device power comparison

<table>
<thead>
<tr>
<th>Inverter systems</th>
<th>Total average SDPR (kVA)</th>
<th>Total peak SDPR (kVA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWM inverter</td>
<td>238</td>
<td>747</td>
</tr>
<tr>
<td>PWM plus boost dc/dc</td>
<td>225</td>
<td>528</td>
</tr>
<tr>
<td>Z-source inverter</td>
<td>199</td>
<td>605</td>
</tr>
</tbody>
</table>

The Z-source inverter’s average SDPR is the smallest among the three, while the conventional PWM inverter’s SDPRs are the highest in both average and peak values. The average SDPR also indicates thermal requirements and conversion efficiency.

2.4.2 Actual Price Comparison

The SDPR numbers above are the theoretical ratings of semiconductors required by the three inverter topologies. However, the commercially available integrated power modules (IPMs) [or insulated gate bipolar transistors (IGBTs)] are limited in terms of voltage and current ratings. Assuming that the voltage rating is limited to 600 V, and the current rating is chosen to be two times the average current stress of each inverter topology, the selected devices and price quotes from a distributor are listed in Table 2.2. Again, the Z-source has the lowest price among the three inverters. In addition, because it has fewer components, a higher mean time between failures can be expected, which leads to better reliability.

Table 2.2. Actual price comparison

<table>
<thead>
<tr>
<th>Inverter systems</th>
<th>Selected devices</th>
<th>Total price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Number of pieces)</td>
<td></td>
</tr>
<tr>
<td>PWM inverter</td>
<td>PM400DSA060, 600V/400A dual pack (3)</td>
<td>$269.60*3 = $808.80</td>
</tr>
<tr>
<td>PWM plus boost dc/dc</td>
<td>PM400DSA060, 600V/400A dual pack (1) plus PM200CL060, 600V/200A 6 pack (1)</td>
<td>$240+$269.60 = $509.60</td>
</tr>
<tr>
<td>Z-source inverter</td>
<td>PM300CL060, 600V/300A 6 pack (1)</td>
<td>$308.88</td>
</tr>
</tbody>
</table>
2.4.3 Passive Components Comparison

The inverter cost mainly includes the semiconductors, passive components, and control circuit. A universal DSP board and a gate drive board for 6 switches are enough to control the Z-source inverter, therefore the controller board cost should be the same as a traditional PWM inverter and lower than the dc/dc boost inverter because the Z-source has the least component count, thus requiring least number of gate drive circuits, power supplies, and communication connections.

Passive components are designed according to switching frequency and their current ripple and voltage ripple requirements. For the conventional PWM inverter case, the maximum capacitor voltage ripple is

$$\Delta V_c = \frac{P_o}{V_iC} \left(1 - \frac{3}{4}M\right), \quad (2.7)$$

where $T_s$ is the switching cycle.

The maximum voltage ripple across the capacitor in the dc/dc boost plus PWM inverter is

$$\Delta V_c = \frac{P_o}{V_iC} (1 - D) - \cos \varphi \frac{4P_o T_s}{3V_{DC}MC} \left(\frac{3}{4}M - D\right). \quad (2.8)$$

The current ripple through the inductor of the dc/dc converter is

$$\Delta I_L = \frac{V_i}{L} DT_s. \quad (2.9)$$

The voltage ripple across the capacitor in the Z-source inverter is

$$\Delta V_c = \frac{P_o}{V_iC} \left(1 - \frac{\sqrt{3}}{2}M\right) T_s. \quad (2.10)$$

The current ripple through the inductor is

$$\Delta I_L = \frac{V_i \sqrt{3M}}{2L(\sqrt{3M} - 1)} \left(1 - \frac{\sqrt{3}}{2}M\right) T_s. \quad (2.11)$$

The rms current ripple through the capacitors is calculated by the MATLAB programs in Appendix B.

An example of required passive components at input power of 50 kW is shown in Table 2.3.
based on the following requirements:

- Current ripple through inductors <10%
- Voltage ripple across capacitors <3%
The two inductors can be made using one magnetic core (to be discussed in Section 3), which will minimize the total size. As seen in Table 2.3, the passive components requirement of the Z-source inverter is similar to or slightly higher than that of the dc/dc–boosted PWM inverter.

Table 2.3. Required passive components

<table>
<thead>
<tr>
<th>Inverter systems</th>
<th>Number of inductors</th>
<th>Inductance (μH)</th>
<th>Average inductor current (A)</th>
<th>Number of capacitors</th>
<th>Capacitance (μF)</th>
<th>Capacitor rms ripple current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional PWM inverter</td>
<td>0</td>
<td>N/A</td>
<td>N/A</td>
<td>1</td>
<td>667</td>
<td>106</td>
</tr>
<tr>
<td>dc/dc boost + PWM</td>
<td>1</td>
<td>510</td>
<td>200</td>
<td>1</td>
<td>556</td>
<td>124</td>
</tr>
<tr>
<td>Z-source inverter</td>
<td>2(1)</td>
<td>384</td>
<td>200</td>
<td>2</td>
<td>420</td>
<td>115</td>
</tr>
</tbody>
</table>

2.4.4 Efficiency Comparison

Efficiency is an important criterion for any power converter. High efficiency can reduce thermal requirements and cost. An efficiency comparison is conducted based on the following conditions: the conventional inverter is always operating at a modulation index of 1, the dc/dc boost plus PWM inverter boosts the dc voltage to 420 V, and the Z-source inverter outputs the maximum obtainable voltage while keeping the switch voltage under 420 V. The same motor model is used to calculate the motor loss. Switching devices are selected for each inverter topology to calculate their losses. The operation conditions are listed in Table 2.4.

Table 2.4. Operation conditions at different power

<table>
<thead>
<tr>
<th>Power rating</th>
<th>50 kW</th>
<th>40 kW</th>
<th>30 kW</th>
<th>20 kW</th>
<th>10 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>56 kVA</td>
<td>47 kVA</td>
<td>38 kVA</td>
<td>27 kVA</td>
<td>14 kVA</td>
</tr>
<tr>
<td>Fuel cell voltage (V)</td>
<td>250</td>
<td>280</td>
<td>305</td>
<td>325</td>
<td>340</td>
</tr>
<tr>
<td>Motor current (A)</td>
<td>Conventional PWM inverter</td>
<td>209.4</td>
<td>158.5</td>
<td>115.9</td>
<td>77.3</td>
</tr>
<tr>
<td></td>
<td>dc/dc boost + PWM inverter</td>
<td>124.7</td>
<td>105.6</td>
<td>84.2</td>
<td>59.9</td>
</tr>
<tr>
<td></td>
<td>Z-source inverter</td>
<td>129.5</td>
<td>105.3</td>
<td>81.1</td>
<td>56.2</td>
</tr>
</tbody>
</table>
The selected devices are as follows:

The switches for the main inverters are FUJI IPM 6MBP300RA060; the switch for the dc/dc boost converter is FUJI 2MBI 300N-060.

The calculated efficiencies of inverters [11], as well as of the inverter plus the motor, are listed in Tables 2.4 and 2.5, respectively. The calculation results are also shown in Figs. 2.4 and 2.5, respectively.

Based on the comparison below, the Z-source inverter provides the highest efficiency.

<table>
<thead>
<tr>
<th>Table 2.5. Inverter efficiency comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Inverters</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>10 kW</td>
</tr>
<tr>
<td>20 kW</td>
</tr>
<tr>
<td>30 kW</td>
</tr>
<tr>
<td>40 kW</td>
</tr>
<tr>
<td>50 kW</td>
</tr>
<tr>
<td>Conventional PWM inverter</td>
</tr>
<tr>
<td>0.968</td>
</tr>
<tr>
<td>0.968</td>
</tr>
<tr>
<td>0.968</td>
</tr>
<tr>
<td>0.966</td>
</tr>
<tr>
<td>0.964</td>
</tr>
<tr>
<td>dc/dc boost + PWM inverter</td>
</tr>
<tr>
<td>0.964</td>
</tr>
<tr>
<td>0.966</td>
</tr>
<tr>
<td>0.966</td>
</tr>
<tr>
<td>0.965</td>
</tr>
<tr>
<td>0.964</td>
</tr>
<tr>
<td>Z-source inverter</td>
</tr>
<tr>
<td>0.973</td>
</tr>
<tr>
<td>0.973</td>
</tr>
<tr>
<td>0.973</td>
</tr>
<tr>
<td>0.971</td>
</tr>
<tr>
<td>0.969</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2.6. Inverter-motor system efficiency comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Inverters</td>
</tr>
<tr>
<td>10 kW</td>
</tr>
<tr>
<td>20 kW</td>
</tr>
<tr>
<td>30 kW</td>
</tr>
<tr>
<td>40 kW</td>
</tr>
<tr>
<td>50 kW</td>
</tr>
<tr>
<td>Conventional PWM inverter</td>
</tr>
<tr>
<td>0.925</td>
</tr>
<tr>
<td>0.887</td>
</tr>
<tr>
<td>0.846</td>
</tr>
<tr>
<td>0.795</td>
</tr>
<tr>
<td>0.726</td>
</tr>
<tr>
<td>dc/dc boost + PWM inverter</td>
</tr>
<tr>
<td>0.936</td>
</tr>
<tr>
<td>0.917</td>
</tr>
<tr>
<td>0.902</td>
</tr>
<tr>
<td>0.890</td>
</tr>
<tr>
<td>0.880</td>
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<tr>
<td>Z-source inverter</td>
</tr>
<tr>
<td>0.949</td>
</tr>
<tr>
<td>0.930</td>
</tr>
<tr>
<td>0.913</td>
</tr>
<tr>
<td>0.896</td>
</tr>
<tr>
<td>0.877</td>
</tr>
</tbody>
</table>

2.4.5 CPSR Comparison

CPSR is limited mainly by the available dc voltage of the PWM inverter. Fuel cell voltage decreases as the current drawn increases, greatly limiting the motor’s power output and efficiency at high speed. For a conventional PWM inverter, the fuel cell voltage is the dc voltage of the inverter, which drops to 250 V at 200 A. From the 250-V dc voltage, the conventional PWM inverter can yield only 153 V to the motor with a modulation index of 1. This low motor voltage limits CPSR and lowers mechanical output power and efficiency. A PWM inverter with dc/dc
boost can keep the inverter dc voltage constant at 420 V, which in turn increases CPSR by 1.68
times. Theoretically, the Z-source inverter can output whatever voltage level is wanted. To make
the comparison fair, the same device voltage limit is applied; i.e., the maximum voltage across the
device is limited to 420 V. The obtainable output voltage is 237 V; thus CPSR is increased by 1.55
times compared with the traditional PWM inverter. In other words, the motor voltage produced by
the inverters is 1.55 times that produced by the conventional PWM inverter. That means the same
motor can output 1.55 times the power driven by a conventional PWM.

![Inverter efficiency graph](image1)

**Fig. 2.4. Calculated efficiency of inverters.**

![System Efficiency graph](image2)

**Fig. 2.5 Calculated efficiency of inverters plus motor.**
2.5 Simulation Results

To verify the validity of these comparisons, simulation models for the three inverters have been developed. As an example, simulation results at 30 kW are given in Fig. 2.6.

(a) Switch voltage, current and output voltage, current of conventional PWM inverter.

(b) Boost converter and inverter switch currents, voltage and output voltage, currents of dc/dc boost PWM inverter.
Capacitor voltage, inductor current of Z network, switch current and voltage, and output current and voltage of Z-source inverter.

Fig. 2.6. Simulation results of different inverters at 30 kW.

Through simulation results and the models developed, we have confirmed the validity of the comparisons performed. For example, the output current of the traditional PWM inverter is much higher than that of the other two cases, which means higher inverter losses, higher current device needed, and higher current to the motor. The obtainable output power from the motor is greatly limited by the dc voltage for the conventional PWM inverter.

2.6 Summary

A comprehensive comparison of the three inverter systems has been performed and reported. The comparison results show that the Z-source inverter can increase inverter conversion efficiency by 1% over the two existing systems and inverter-motor system efficiency by 2 to 15% over the conventional PWM inverter. The Z-source also reduces the SDPR by 15%, which leads to cost reduction. Moreover, the CPSR is greatly extended (1.55 times) compared with the system driven by the conventional PWM inverter. Thus the Z-source inverter system can minimize stresses and motor size and increase output power greatly. Along with these promising results, the
Z-source inverter offers a simplified single-stage power conversion topology and higher reliability because shoot-through can no longer destroy the inverter. The existing two inverter systems suffer the shoot-through reliability problem. In summary, the Z-source inverter is very promising for this application.
3. Design of the 55-kW Z-Source Inverter for Fuel Cell Vehicles

This section presents a detailed design process of the 55 kW Z-source inverter for FCVs. The schematic of the main circuit is shown in Fig. 3.1.

![Schematic of the Z-source inverter system.](image)

Fig. 3.1. Schematic of the Z-source inverter system.

3.1 Circuit Design

3.1.1 Specifications

Fuel cell voltage: 250 V @ 55 kW and 400 V at no load

Output power: 55 kW peak for 18 seconds and 30 kW continuous

The components to be designed or selected are the two inductors, L₁ & L₂; the three capacitors, C₁, C₂, and C₃; the inverter switches; and the diode, D.

3.1.2 Inductor Design

During traditional operation mode, when there is no shoot-through, the capacitor voltage is always equal to the input voltage; therefore, there is no voltage across the inductor and only a pure dc current going through the inductors. The purpose of the inductors is to limit the current ripple through the devices during boost mode with shoot-through. During shoot-through, the inductor current increases linearly, and the voltage across the inductor is equal to the voltage across the capacitor; during non-shoot-through modes (six active modes and the two traditional zero modes), the inductor current decreases linearly and the voltage across the inductor is the difference between the input voltage and the capacitor voltage. The average current through the inductor is

\[ \bar{I}_L = \frac{P}{V_{in}}, \]  

where \( P \) is the total power and \( V_{in} \) is the input voltage.
The average current at 55 kW at 250 V input is

\[ I_L = \frac{55000}{250} = 220 \text{ A.} \]  

(3.2)

The maximum current through the inductor occurs when the maximum shoot-through happens, which causes maximum ripple current. In our design, 30% (60% peak to peak) current ripple through the inductors during maximum power operation was chosen. Therefore, the allowed ripple current was 132 A, and the maximum current through the inductor was 286 A. A 600-V device was chosen; therefore, the circuit was designed to operate at the maximum voltage across the switches, 400 V. The maximum shoot-through duty cycle can be calculated by

\[ \frac{1}{1-2D_0} = \frac{400}{250}. \]

\[ D_0 = 0.1875. \]  

(3.3)

For a switching frequency of 10 kHz, the shoot-through time per cycle is 18.75 \(\mu\)s. The capacitor voltage during that condition is

\[ V_c = \frac{250 + 400}{2} = 325V. \]  

(3.4)

To keep the current ripple less than 120 A, the inductance must be no less than

\[ \frac{18.75 \times 325}{132} = 46.2 \mu H. \]  

(3.5)

![Fig. 3.2. Coupled inductors.](image-url)
To minimize the size and weight of the inductors, the two inductors are built together on one core, as shown in Fig. 3.2. For a single coil on one core, the flux through the core is

$$\phi = PN_i,$$  \hspace{1cm} (3.6)

where $P$ is a constant related to the core material and dimension, $N$ is the number of turns of the coil, and $i$ is the current through the coil. The inductance of the coil is

$$L = \frac{N\phi}{i} = PN^2.$$  \hspace{1cm} (3.7)

For the two inductors in the Z-source inverter, because of the symmetry of the circuit, the current through the inductors is always exactly the same. For two coils on one core with exactly the same current, $i$, the flux through the core is

$$\phi = 2PN_i.$$  \hspace{1cm} (3.8)

The resulting inductance of each coil when supplying exactly the same current to the two coils is

$$L = \frac{N\phi}{i} = 2PN^2.$$  \hspace{1cm} (3.9)

The inductance of each coil is doubled. Therefore, equivalently, we need to build two coils with $23.1 \, \mu H/286 \, A$ each on one core; or, say, one coil with $23.1 \, \mu H/572 \, A$. A Metglas AMCC_250 core was selected to reduce the loss.

Choosing maximum $B=1.2 \, T$ at peak current,

$$I_{\text{peak}} = 440 \ast 1.3 = 572 \, A,$$  \hspace{1cm} (3.10)

$$N = \frac{LI}{BS} = 9.72,$$  \hspace{1cm} (3.11)

where $S$ is the area of the core, which is $11.4 \, \text{cm}^2$, we take 10 turns.

To check the design, the magnetization curves of AMCC_250 are shown in Fig. 3.3. The selected magnetizing force at peak power is

$$MF = 572 \ast 10 = 5720AT.$$  \hspace{1cm} (3.12)
Fig. 3.3. Magnetizing curve of the selected core.

From the curve, we choose an air gap of 6.0 mm to prevent the core from getting into saturation. The inductance then will be

\[ L = 10^2 \times 0.23 = 23 \mu H , \]  

which is satisfactory for our design.

The winding is related to the copper loss; thus we can design the winding rated at the continuous power of 30 kW at 305 V input, and the current through the inductor is

\[ I_{av} = \frac{30000}{305} = 98.4 A . \]  

With two coils in parallel, the average current is 196.8A. Using six of the wires 380/33G, which are rated at 40 A, the wire area is

\[ S = \pi r^2 = 3.14 \times 2.45^2 = 18.8 mm^2 . \]  

The total area for the winding is

\[ S_f = 10 \times 6 \times 18.8 = 1128 mm^2 . \]  

The window utilization factor is

\[ f = \frac{1128}{90 \times 25} = 50.1\% . \]
3.1.3 Capacitor Selection

The purpose of the capacitor is to absorb the current ripple and maintain a fairly constant voltage so as to keep the output voltage sinusoidal. During shoot-through, the capacitor charges the inductors, and the current through the capacitor equals the current through the inductor. Therefore, the voltage ripple across the capacitor can be roughly calculated by

\[ \Delta V_C = \frac{I_{av} T_0}{C}, \]

where \( I_{av} \) is the average current through the inductor, \( T_0 \) is the shoot-through period per switching cycle, and \( C \) is the capacitance of the capacitor. To limit the capacitor voltage ripple to 3% at peak power, the required capacitance is

\[ C = \frac{200 \times 18.75 \mu F}{325 \times 3\%} = 384.6 \mu F. \]

Another function of the capacitor is to absorb the ripple current. As discussed in Section 2, a MATLAB program was provided to calculate the ripple current. The power factor of the load is a necessary value for the program. For induction machines, the power factor at high power is usually fairly high, so 0.9 was used for the calculation. Using these numbers, the rms ripple current through the capacitor was 111 A at peak power. Electronic Concepts UL31 500-V/200-uF film capacitors were selected, with two connected in parallel to form one conceptual capacitor in the Z-source inverter. The reason for choosing a film capacitor instead of an electrolytic capacitor was to reduce the size and enhance the performance. The fuel cell is a double-layer capacitor by itself, so theoretically no capacitor is needed in parallel with it. However, to minimize the high-frequency current path, one UL31 was used in parallel with the fuel cell (C3).

3.1.4 Device Selection

The maximum voltage across the switches and the diode was 400 V. The peak current through the switches occurred at the peak power of 55 kW. At maximum power, the output voltage was boosted to the maximum voltage, with the limit of the device PN voltage no higher than 400 V. Within this limit, we can calculate the modulation index of the inverter at maximum power and 250 V input, using maximum constant boost control, by

\[ \frac{1}{\sqrt{3}M - 1} \times 250 = 400, \]

\[ M = 0.938 \]
The output obtainable voltage is

\[ V_{\text{load}} = 400 \times 0.938 \times \frac{1.732}{2.828} = 230V. \]  

(3.21)

The load current can be calculated with the assumption that the power factor at maximum power is 0.9:

\[ I_{\text{load}} = \frac{55000}{\sqrt{3} \times 0.9 \times 230} = 153.5. \]  

(3.22)

As discussed in Section 2, the maximum current through the switches is

\[ I_s = \frac{1}{2} I_{\text{load}} + \frac{2}{3} I_L. \]  

(3.23)

Using Eq. (3.23), the maximum current through the switches is 280 A. The average current through the diode equals the average current through the inductor, which is 100 A for continuous power. The peak current through the diode is twice the inductor current, which occurs during traditional zero states; therefore, the peak current through the diode is 572 A. Therefore, the following devices were selected, considering the high temperature requirement: a 600-V/600-A six-pack IPM PM600CLA060 for the inverter bridge, and two 600-V/600-A diode QRS0660T30s in parallel for the input diode.

### 3.2 dc Rail Clamp Circuit

The selected IPM turned out to be three dual-pack IPMs in parallel with three different P and N pairs not internally connected (this layout was not indicated in the data sheet and was discovered after the IPM arrived). This layout required external connection and voltage overshoot suppression. To reduce the voltage overshoot across the device, a dc rail clamp circuit was implemented. The 3 P-N pairs for each dual-pack IPM were connected externally (as should have been done inside the module) via a busbar, and each P-N pair had its own clamping circuit to make the circuit path as small as possible. The circuit is shown in the dotted part of Fig. 3.4. One small capacitor, C₆, was put across P and N directly, along with two capacitors, C₄ and C₅, and one diode, D₁, all connected in series, as shown in Fig. 3.4(a). Another two small diodes (TO 247 package), D₂ and D₃, in series were connected to the main power circuit from the clamp circuit, forming a discharge loop for C₄. As seen in Fig. 3.4(b), when the current to the inverter, Iᵢ, has a step change, the dc rail clamping circuit provides an extra absorbing path for the extra current maintained by the parasitic inductance of the main bus-bar, helping to reduce the overshoot voltage across the device. C₆ can be easily discharged to C₁ and C₂ through the inductors.
Fig. 3.4(c) shows the two paths for discharging the other two capacitors, $C_4$ and $C_5$, in the clamping circuit. Looking at Fig. 3.4(c), $C_4$ can be discharged through $C_2$, $C_3$, $D_2$, and $D_3$; and $C_5$ can be discharged to $C_1$. The series connection of $D_1$, $D_2$, and $D_3$ is in parallel with the main diode $D$. The forward voltage drop of the main diode $D$ is relatively higher than the low-current diodes. The reason to put $D_2$ and $D_3$ in series is to ensure that most of the steady state input current goes through the main diode $D$ instead of $D_1$, $D_2$, and $D_3$. The capacitor $C_6$ will cause extra loss during shoot-through; therefore, the capacitance of this capacitor has to be very small. The actual part numbers are

- $D_1$: two Fairchild RHRP3060s in parallel
- $D_2$ and $D_3$: Fairchild THRG80100
- $C_4$ and $C_5$: AVX SK097C105MAA
- $C_6$: Orange drop 715P600V103J
(a) The schematic of the dc-rail clamping circuit (dashed part).

(b) Charging loop.

(c) Discharging loop.

Fig. 3.4. The dc-rail clamping circuit.
3.3 Gate Drive Board, Sensor Board, and DSP Control Board

The gate drive board for the IPM takes 15-V dc input and gating signals from the DSP board through fiber optics. It consists of six isolated power supplies and six channels of gate control. The detailed schematic of the gate drive board is provided in Appendix C.

Two output current sensors, one inductor current sensor, one input voltage sensor, and one capacitor voltage sensor ($C_2$) were provided to enable further control and protection functions. The protection function provided included

- Overload current protection: The inverter trips when the load current is over 300A.
- Over-input current protection: The inverter trips when the input current (inductor current) is over 230A.
- Over-input voltage protection: The inverter trips when the input voltage is over 400 V.
- Over-device voltage protection: The inverter trips when the voltage across the device is over 420V.

A universal DSP control board previously developed at MSU is used to control the Z-source inverter. DSP control requirements of the Z-source inverter should not differ from the traditional PWM inverter’s. The universal DSP control board employs fiber optics for communication to the gate drive boards, which is not necessary.

3.4 Thermal and 3-D Design

3.4.1 Thermal Design

Early on, it was realized that the hottest part of the inverter was going to be the Powerex 600-V/600-A six-pack IPM PM600CLA060; so the thermal design focused mostly on the temperature increase for this part. The junction-to-case thermal resistance per IGBT is 0.07°C/W, making the Rth of the IPM 0.01167°C/W. The case-to-fins Rth of the IPM is 0.014°C/W, making the total Rth of the IPM 0.02567°C/W. The minimum over-temperature point of the IPM is 135°C, but the reset temperature is 125°C. We will assume 125°C is our maximum operating temperature.

During 30-kW operation, during non-boost operation, the loss is calculated at 700 W; at 55 kW, while boosting, the loss is calculated as 1.350 kW. Assuming that the 55-kW condition is used for a long enough time that the IPM temperature increase levels off to a steady state value, the 1.350-kW loss is used as our worst-case condition. A 1.350-kW loss translates into a fin temperature of 90.35°C, given a junction temperature of 125°C.

The heat sink used is a 10.5×8×0.75-in.-thick water-cooled heat sink with copper inserts.
placed above the copper tubing to increase thermal conductivity. Figure 3.5 shows the heat sink with the IPM mounted to indicate how much of the heat sink the IPM takes up. The thermal resistance for the whole heat sink is 0.005°C/W with a 1.5-gal-per-min flow rate. Since the IPM covers approximately half the heat sink, 0.01°C/W is used as the thermal resistance that the IPM sees. This gives a maximum water temperature of 76.85°C, not taking into account the increase in water temperature from the inlet to the last part of the tubing underneath the IPM.

The temperature rise of the flowing coolant is governed by Eq. (3.24):

$$\Delta T = \frac{\text{Power}(\text{J/s})}{(\text{Specific Heat} \times \text{Mass}(\text{g/s}))}$$

A 1:1 mix of water and ethylene glycol has a specific heat of 3.56 J/g°C and a density of 3800 g/gallon. The temperature rise of the water at the end of the IPM is then

$$\Delta T_{\text{after IPM}} = 4.46^\circ C \times \frac{L}{F},$$

with $L$ as the loss in kW and $F$ as the flow in gal/min. At 1.5 gal/min and 1.35 kW of loss, the water temperature rise is 4°C.

If we assume that the thermal resistance of the heat sink takes into account the water temperature rise, then the maximum inlet water temperature is 76.85°C. If it is found that the thermal capacitance of the heat sink is large enough that the amount of time that the inverter runs at 55 kW does not raise the temperature to a steady state, the maximum temperature obviously decreases. As a lower limit, consider the maximum water temperature at 700 W, the 30-kW condition. Using the same method, the maximum inlet temperature is found to be 100°C.

To get an idea of the thermal capacitance associated with aluminum/copper under the IPM, some basic numbers can be run. The heat sink is mostly aluminum with some copper; we can assume the water to be a constant temperature. For simplicity, assume that the whole unit is made of aluminum and that half the total heat sink size will affect the IPM. Thermal capacitance is found by Eq. (3.25):

$$C_{\text{th}} = \nu \times \rho \times C_t,$$

with $\nu$ as volume in m³, $\rho$ as density in kg/m³, and $C_t$ as specific heat in J/kg°C. Aluminum’s properties are 2710 kg/m³ and 875 J/kg°C, and the total applicable heat sink area is $3.687 \times 10^{-4}$ m². This gives a thermal capacitance of 385 J/°C. With the thermal resistance at 0.01, the thermal time constant is about 4 seconds. As per the specifications of 55-kW output for 18 seconds, the
temperature will have reached steady state.

Taking into account the copper strips inserted onto the top of the heat sink does increase the thermal time constant by 1.5 seconds; however, this does little to affect the final result.

3.4.2 Part Placement

As stated earlier, the thermal considerations of the IPM recommend that the IPM be placed as close to the inlet as possible to get the coolest water. Placement of the input side diodes is based upon the total circuit length of the series connection of all three capacitors and the input side diode. The current design uses a single-diode module as one conceptual diode. For stray inductance and high-frequency noise purposes, only one of these diodes needs to be considered for total circuit length. Thus one diode should be very close to the IPM, while the other can be placed where convenient. The inductor is placed on as much of the copper inserts as possible to help in cooling the core, but its placement on the heat sink is also arbitrary. Figure 3.5 shows the current placement of the IPM, diodes, and inductor.

3.4.3 Busbar Design

Busbar design is important to minimize stray circuit inductance, which minimizes semiconductor device overshoot. For this design, the busbars also support the weight of the capacitors, so copper thickness is a function of both current carrying capability and strength. The busbars have gone through two designs, and a third design was drawn up for use with another IPM module that never needed to be used. Some important features of the current busbar assembly, especially compared with the first iteration, are the addition of external shorting bars for the P and N of the IPM, an upgrade to laminated busbars instead of posts to connect the IPM and diodes to the main busbar assembly 2 in. above the IPM connection point, and easier assembly overall.
Figure 3.6 shows a view of the P and N shorting and connecting bars for the IPM and the diode connecting bars, along with a view of the original design that used copper posts. Using laminated busbars eliminated large circuit loops that added undesired stray inductance, eased assembly since the bars could be put on before the rest of the assembly, and added overall strength to the busbar assembly.

Figure 3.7 shows a schematic of the Z-source inverter with color-coded nodes, along with the 3-D rendering of the inverter with all the busbars included.
3.4.4 dc Clamping Circuit with Heat Sink

Construction of the dc rail clamping circuit is fairly straightforward because it is just two capacitors and two diodes connected in series to act as one larger diode. The problem we faced was both keeping each diode relatively cool and keeping them at the same temperature, since diodes have a negative temperature coefficient. Because the IPM module is constructed like three dual modules with non-connected PN pairs, each pair has its own local clamping circuit for minimum circuit path length. Figure 3.4 (a) shows the Z-source schematic with clamping circuit and the model of the clamping circuits and heat sink.
As seen in Fig. 3.4(a), the clamping circuit diode is in parallel with the main circuit diode, with two extra diodes in series with it to reduce the main circuit current that would otherwise flow through the small clamping circuit diodes. Even with these extra diodes in series with it, some main circuit current is flowing through the clamping circuit diode, if only an amp or two. Along with the current spikes associated with stray inductance energy that is discharged through the clamping circuit during switch turn-off transients, the total current that does flow through the clamping circuit diodes is significant enough to require adequate heat sinking.

**3.4.5 Sensor Boards**

For protection and control purposes, the Z-source inverter prototype includes two voltage sensors and three current sensors. The two voltage sensors measure the input voltage and the voltage on one of the Z-network capacitors. Two current sensors measure two phases of output current, while another current sensor measures the inductor current, which is basically the average input current.

The current sensor board is attached to two output bars constructed to hold the sensor board about an inch above the IPM (Fig. 3.9). Each output bar has a side platform that can be used to glue the current sensor board to it with RTV silicone or some other flexible adhesive. The voltage sensor board was sized specifically to be glued sideways to the middle capacitor, so as not to add height to the inverter.
The Z-source inverter has been successfully constructed with an emphasis on thermal management of the IPM and the clamping circuit diodes. The busbar assembly underwent a fairly major redesign to change the connection points that join the IPM and main circuit diode modules and the capacitors from copper posts to laminated busbars. This change helped to minimize stray inductance of the main noise circuit path, added overall strength, and eased assembly. Figure 3.10 shows the final version of the Z-source inverter prototype as used in testing.
3.5 Summary

The detailed design process of the inverter is provided, including the inductor design, capacitor and device selection, dc rail clamp circuit design, and gate drive board and sensor board design. The 3-D structure design and thermal design are also provided.
4. Shoot-Through PWM Control

Several control methods have been proposed: simple control [8], maximum boost control [12], and maximum constant boost control [10]. In our design, to minimize the size of the inductor, the inductance was selected to be 50 μH; therefore, maximum constant boost was the most suitable control method to minimize current ripples. The original method presented in the paper [10] increases the equivalent frequency for the inductor side, but it also increases the real switching frequency. To minimize the switching loss, a modified PWM method that achieves maximum constant boost and minimum switching loss was proposed and implemented in the prototype.

4.1 Control Methods for Z-Source Inverter

Compared with a traditional voltage source inverter, the Z-source inverter has an extra switching state: shoot-through. During the shoot-through state, the output voltage to the load terminals is zero, the same as traditional zero states. Therefore, to maintain sinusoidal output voltage, the active-state duty ratio has to be maintained and some or all of the zero states turned into shoot-through state.

4.1.1 Simple Control

The simple control [8] uses two straight lines to control the shoot-through states, as shown in Fig. 4.1. When the triangular waveform is greater than the upper envelope, $V_p$, or lower than the bottom envelope, $V_n$, the circuit turns into shoot-through state. Otherwise it operates just as traditional carrier-based PWM. This method is very straightforward; however, the resulting voltage stress across the device is relatively high because some traditional zero states are not utilized.

4.1.2 Maximum Boost Control

To fully utilize the zero states so as to minimize the voltage stress across the device, maximum boost control [12] turns all traditional zero states into shoot-through state, as shown in Fig. 4.2. Third harmonic injection can also be used to extend the modulation index range.
Indeed, turning all zero states into shoot-through state can minimize the voltage stress; however, doing so also causes a shoot-through duty ratio varying in a line cycle, which causes inductor current ripple [10]. This will require high inductance for low-frequency or variable-frequency applications.

### 4.1.3 Maximum Constant Boost Control

The sketch map of maximum constant boost control is shown in Fig. 4.3. This method achieves maximum boost while keeping the shoot-through duty ratio always constant; thus it results in no line frequency current ripple through the inductors. The sketch map of maximum constant boost control with third harmonic injection is shown in Fig. 4.3(b). With this method, the inverter can buck and boost the voltage from zero to any desired value smoothly within the limit of the device voltage.
4.1.4 Voltage Stress Comparison of the Control Methods

To examine the voltage stress across the switching devices, an equivalent dc voltage is introduced. The equivalent dc voltage is defined as the minimum dc voltage needed for the traditional voltage-source inverter to produce the same output voltage. The ratio of the voltage stress to the equivalent dc voltage represents the cost that Z-source inverter has to pay to achieve voltage boost.

The ratios of the voltage stress to the equivalent dc voltage, $k_{stress}$, for the simple control, maximum boost control, and maximum constant boost control are summarized as follows:

\[
\begin{align*}
Gk_{stress} & = 2 - \frac{1}{G}, \quad \text{for simple control} \\
Gk_{stress} & = \frac{3\sqrt{3}}{\pi} - \frac{1}{G}, \quad \text{for maximum boost} \\
Gk_{stress} & = \sqrt{3} - \frac{1}{G}, \quad \text{for maximum constant boost}
\end{align*}
\]

where $G$ is the voltage gain defined as

\[
G = \frac{\hat{v}_o}{V_{dc}/2}, \tag{4.4}
\]

where $\hat{v}_o$ is the peak output phase voltage and $V_{dc}$ is the input voltage to the Z-source inverter. The comparison is shown in Fig. 4.4. In the figure, the voltage stress of simple control is highest among the three, and the maximum boost achieves the minimum voltage stress. However, the maximum boost suffers from the six time load frequency current ripple through the inductor;
therefore, the maximum constant boost control is the most suitable method for our application. Also, the maximum constant boost with third harmonic injection seems to be the better one because it can achieve continuous output voltage variation from zero to infinity.

![Voltage stress comparison of different control methods.](image)

**4.2 Modified PWM Control for Shoot-Through**

From Fig. 4.3, the inverter with maximum constant boost control with third harmonic injection shoots through twice in one cycle (triangular waveform cycle); the equivalent frequency to the inductor is doubled, thus reducing the requirement to the inductors. However, it is obvious from Fig. 4.3 that the real switching frequency of the device also doubles, which increases the switching loss.

In traditional PWM control, there is always a zero state after two active states, as shown in Fig. 4.5. There are two types of zero states, Zero 1 and Zero 2; Zero 1 occurs when all upper three switches are turned on, and Zero 2 occurs when all lower three switches are turned on.

![Switching states sequence of traditional PWM control.](image)
zero states (Zero 1 or Zero 2) into shoot-through state and leave the active states unchanged. The
duty ratio of that shoot-through state equals the shoot-through duty ratio of maximum constant
boost control, which is

\[ D_0 = 1 - \frac{\sqrt{3}}{2} M. \quad (4.5) \]

Therefore, the shoot-through period lasts \((1 - \frac{\sqrt{3}}{2} M)T_s\) in each switching cycle \(T_s\), which means
that the zero state (Zero 1 or Zero 2) turned into shoot-through state lasts \((1 - \frac{\sqrt{3}}{2} M)T_s\). To realize
this function, there are two possible schemes, shown in Fig. 4.6(a) and (b) respectively.

For the case in (a), assume the reference signals in traditional SPWM are

\[ \begin{align*}
  v_a &= M \sin(\omega t) \\
  v_b &= M \sin(\omega t - \frac{2}{3} \pi) \\
  v_c &= M \sin(\omega t - \frac{4}{3} \pi)
\end{align*} \]  

(4.6)

Fig. 4.6. Modified PWM scheme.
There are three different intervals in one line cycle in the modified PWM method. The reference signals in the three intervals are, respectively,

\[ v_a' = \sqrt{3}M - 1 \]
\[ v_b' = \sqrt{3}M - 1 + v_b - v_a \]
\[ v_c' = \sqrt{3}M - 1 + v_c - v_a \]

\[ v_a' = \sqrt{3}M - 1 + v_a - v_b \]
\[ v_b' = \sqrt{3}M - 1 \]
\[ v_c' = \sqrt{3}M - 1 + v_c - v_b \]

\[ v_a' = \sqrt{3}M - 1 + v_a - v_c \]
\[ v_b' = \sqrt{3}M - 1 + v_b - v_c \]
\[ v_c' = \sqrt{3}M - 1 \]

where \( v_a, v_b, \) and \( v_c \) are expressed in Eq. (4.6). When the triangular waveform is higher than the maximum value of the three, which is \( \sqrt{3}M - 1 \), the circuit is turned into shoot-through state by gating on all the switches. Otherwise, it operates the same as traditional synchronized PWM (SPWM). It is obvious that the duration of each active state in a switching cycle is kept the same as in traditional SPWM by keeping the distance between the reference curves unchanged; therefore, the fundamental element of the output voltage will still be kept sinusoidal.

The second case is shown in Fig. 4.6(b), where again there are three different intervals in one line cycle for the modified PWM. The reference signals in the three intervals are, respectively,

\[ v_a' = 1 - \sqrt{3}M \]
\[ v_b' = 1 - \sqrt{3}M + v_b - v_a \]
\[ v_c' = 1 - \sqrt{3}M + v_c - v_a \]

\[ v_a' = 1 - \sqrt{3}M + v_a - v_b \]
\[ v_b' = 1 - \sqrt{3}M \]
\[ v_c' = 1 - \sqrt{3}M + v_c - v_b \]

\[ 2k\pi + 7\pi \frac{6}{6} \leq \omega t < 2k\pi + \frac{11}{6}\pi, \quad k = 0,1,2,\ldots \quad (4.10) \]

\[ 2k\pi + 5\pi \frac{6}{6} \leq \omega t < 2k\pi + \frac{3}{2}\pi, \quad k = 0,1,2,\ldots \quad (4.11) \]
where $v_a$, $v_b$, and $v_c$ are expressed in Eq. (4.6). When the triangular waveform is lower than the minimum value of the three, $1 - \sqrt{3}M$, the inverter is turned into shoot-through state by gating on all the switches. Otherwise, it operates the same as traditional SPWM. Also, the duration of each active state in a switching cycle is kept the same as in traditional SPWM; therefore, the output waveform will still be kept sinusoidal.

As shown in Fig. 4.6, all six switches turn on and off seven times in one cycle in total, which reduces the switching actions quite significantly. The equivalent frequencies to the inductors and to the load are both the triangular frequency.

The two different schemes achieve the same effect for the boost function of the inverter. But the switching actions and the on time of the upper and lower switches in a phase leg are different. As seen in Fig. 4.6(a), all upper switches are turned on for $1/3$ of the line cycle in turn, and all the lower switches are turned on and off every switching cycle. It is the opposite for the case shown in (b). This results in an unbalanced switching loss and on-state loss between each of the switches inside the IPM, which could require a lower coolant temperature. To minimize this unbalance, the two methods are alternated every line cycle.

### 4.3 Summary

To reduce the switching loss, two modified PWM schemes are introduced. The PWM scheme may cause uneven loss on the switches in the same leg. Therefore, the two schemes are implemented in an alternating sequence to make the loss on each switch even. By using this method, the average switching frequency of the switches is $7/6$ of the triangular frequency.
5. Experimental Results

5.1 Testing Setup

A testing system as in Fig. 5.1 was set up to test the prototype.

![Z-source inverter testing setup](image)

**Fig. 5.1. Z-source inverter testing setup.**

In this testing system, the Z-source inverter is used to drive a BALDOR motor ZDM4115T. The motor is coupled mechanically with the ACS alternator 2733-G. The output voltage of the alternator is rectified by a rectifier and fed back to the input of the Z-source inverter by a boost converter. In this way, we can control the motor torque by controlling the current fed to the dc input with the boost converter.

The output voltage of the Z-source inverter is a PWM signal. To extract and measure the fundamental component of the output voltage, a filter is used, as shown in Fig. 5.2. This filter can eliminate the switching frequency PWM ripple. The resistor used is for damping the LC resonance.

![Output voltage monitor filter](image)

**Fig. 5.2. Output voltage monitor filter.**

5.2 Testing Results during Normal Operation (No Shoot-through)

As will be discussed in Section 6, the inverter without battery could self-boost when the modulation index or the load power factor is low. During our test, the modulation index was set to 1.15 and load torque was controlled by the dc boost converter connected to the generator. Figure 5.3 shows waveforms of the Z-source inverter during normal operation mode without boost. Figures 5.3(a), (b), and (c) show the results at 10, 20, and 30 kW operation, respectively.
The input voltages were 110, 196, and 305 V, respectively.

Fig. 5.3. Experimental results at different power levels during normal mode without shoot-through.

Based on the experimental results, without shoot-through (or boost), the inverter operates just as a traditional inverter, where the capacitor voltage equals the input voltage. Sinusoidal output voltages after the monitor LC filter and motor current were obtained; they demonstrated
high-fidelity PWM control and low harmonic distortion, which is a result of the zero dead time needed in the Z-source inverter.

### 5.3 Testing Results with Boost Mode

At high-frequency operation, when a higher voltage is required, the inverter turns into boost mode. The experimental results of 40 kW and 50 kW operation are shown in Fig. 5.4. The input voltages of these two conditions were 280 and 250 V, respectively.

![Experimental Results](image)

*Fig. 5.4. Experimental results of 50kW with boost factor of 1.5.*
As seen from the experimental results, the inverter operates in boost mode with shoot-through. In both cases, the PN voltage across the IPM is boosted to around 380 V, thus increasing the output voltage. The modulation index of the PWM for 40-kW conditions is 1.01. The output voltage of the inverter is boosted to 230 V; for a traditional inverter, the available voltage is only 171 V with a modulation index of 1. The modulation index of the PWM for 50-kW conditions is 0.957. The output voltage of the inverter is 218 V line to line, while the obtainable output voltage for a traditional inverter at 250 V input is 153 V with a modulation index of 1. It was successfully demonstrated that the Z-source inverter can greatly boost the output voltage as desired. Also, the motor current is pure sinusoidal, which confirms that the Z-source inverter will produce very low harmonics.

5.4 Inverter Efficiencies

The inverter efficiency was measured during the test at ORNL. There are two types of loads, RL load and a dynamometer. For the RL load in normal operation mode without boost, the modulation index of the inverter is kept constant; and the output voltage increases with the increase of the input voltage. During boost mode, the output voltage is controlled by a potentiometer controlling the modulation index and shoot-through duty ratio. Figure 5.5(a) shows the measured efficiency with the RL load. The point marked with a dark cross is the test point during boost; others are all during normal mode without shoot-through. For the test using the dyno as the load, the output voltage is always kept proportional to the output frequency to keep the magnetizing current constant (or constant V/F control); thus the output power can be changed by adjusting the load torque, adjusting the input voltage, or adjusting the boost factor during boost mode. The inverter efficiency with different input voltages and different power levels is shown in Fig. 5.5(b). Note that the inverter efficiency is defined against the output apparent power of the inverter. The points with dark marks are the testing points with boost, and others are in normal operation modes without shoot-through. The efficiency was above 97% for most operation points, which achieved the 97% efficiency goal. More noticeably, the efficiency became much higher than 97% at medium and low power, which is extremely beneficial to vehicles, according to most driving cycles.
5.5 Comparison of the Prototype to the FreedomCAR Goals

The FreedomCar has a set of specifications and goals for the traction drive inverter to meet. However, it should be noted that the FreedomCar has not yet set up clear goals for the dc-dc boost converter that are used for fuel cell vehicles and HEVs.

The Z-source inverter prototype has both inverter and dc-dc boost converter functions (i.e., equivalent to the traditional two stages of power conversion: inverter plus dc-dc boost converter), thus making comparison difficult. However, a comparison of the prototype to the initial goals of FreedomCar is conducted and showed in Table 5.1. The numbers in the parentheses of the FreedomCar Goals column are the corresponding goals for the combined total system of inverter and dc-dc boost converter, assuming that the goals for the dc-dc boost converter are the same as the inverter. The power density numbers in the parentheses of the Prototype column are based on the actual peak power capacity of 80 kW that was successfully tested at ORNL, while the outside numbers are based on the design value of 55 kW. As can be seen from the table, the most important goals: the efficiency, power, and current ratings are all well met and exceeded. The power density goals are well met by the prototype based on the actual peak power capacity of 80 kW for the combined dc-dc boost converter and inverter system goals. The only problem is the coolant temperature goal of 105°C, which was not possible for this prototype with the semiconductor device’s maximum junction temperature limit of 125°C. Future higher temperature devices and more advanced circuit topology may make this goal possible. A commercially available heatsink is used in our prototype for cost consideration, which contributes
almost half of the inverter weight and a significant portion of the inverter volume. The capacitors used are commercial products, which have a cylinder shape with legs that are difficult for packaging. Significant void space exists in the prototype, thus making the volume unnecessarily larger. The size and the weight of the prototype can be further reduced with custom designed heatsink and capacitor shapes. Once again, all the goals (efficiency, power, and current) that are related to the technology are well met and exceeded. The volume and weight goals that are strongly related to engineering aspects can be met through better engineering and custom designed parts for full utilization of the space.

<table>
<thead>
<tr>
<th>Requirement</th>
<th>FreedomCAR Goals (inverter+dc-dc)</th>
<th>Prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific power @ peak load (kW/kg)</td>
<td>12 (6*)</td>
<td>4.7 (6.8*)</td>
</tr>
<tr>
<td>Volumetric power density (kW/l)</td>
<td>12 (6*)</td>
<td>4.7 (6.8*)</td>
</tr>
<tr>
<td>Efficiency, 10-100% speed, 20% rated torque (%)</td>
<td>97%</td>
<td>&gt;97%</td>
</tr>
<tr>
<td>Peak power (kW)</td>
<td>55 (55*)</td>
<td>80</td>
</tr>
<tr>
<td>Continuous power (kW)</td>
<td>30 (30*)</td>
<td>30</td>
</tr>
<tr>
<td>Maximum current, rms (A)</td>
<td>300 (300*)</td>
<td>300</td>
</tr>
<tr>
<td>Coolant inlet temperature</td>
<td>105 (105*)</td>
<td>77</td>
</tr>
</tbody>
</table>

* indicates the goals for the total combined system of inverter and dc-dc boost converter assuming the dc-dc boost converter has the same goals as the inverter.

▼ shows the prototype power density values based on the actual peak power capacity of 80 kW that was tested, while the value of 4.7 is based on the designed peak power of 55 kW.
6. Z-Source Inverter Self-Boost Phenomenon

The purpose of this project is to verify the Z-source inverter technology and examine the possibility of using it in fuel cell vehicles. The current configuration of the Z-source inverter prototype does not include the dispensable battery and is not able to handle transients. During the testing, the current Z-source inverter without a battery was discovered to have a voltage boost when operated at low speed, low modulation index, and low load power factor without intentional insertion of shoot-through. This self voltage boost was initially deemed a problem. However, theoretical analysis, simulation, and experiments have proved that it can be solved when a battery is incorporated into the Z-source inverter. A further, deeper investigation and discussions with fuel cell control experts revealed that the self voltage boost problem actually is a needed function for faster and more reliable fuel cell startup, especially for freeze startups, as mentioned in Section 1.3. However, this self-boot phenomenon can be a problem for some applications other than FCVs. In addition, this self-boost has to be and can be controlled. Sections 6.1 through 6.3 show the theory, control/solution, and simulation verification of the self-boost phenomenon, respectively.

6.1 Z-Source Inverter Self-Boost

During normal operation, there is no shoot-through. Assuming the inductor current is a pure dc value, for a system shown in Fig. 6.1, the output voltage at modulation index of $M$ is

$$V_{Lab} = \frac{3V_p M}{2\sqrt{2}}. \quad (6.1)$$

Assuming the load current is $I_{Load}$ and the load power factor is $\cos \phi$, the total power of the system is
\[ P = \sqrt{3} V_{Lab} \cdot I_{Load} \cdot \cos \phi = \frac{3V_oM}{2\sqrt{2}} I_{Load} \cdot \cos \phi . \]  \hspace{1cm} (6.2)

The inductor current can be calculated:

\[ I_L = I_{L1} = I_{L2} = \frac{P}{V_o} = \frac{3M \cdot I_{Load} \cdot \cos \phi}{2\sqrt{2}} . \]  \hspace{1cm} (6.3)

The inverter dc side current, \( I_i \), is a pulse signal. The peak value of the current is the maximum load current, which is

\[ \max(I_i) = \sqrt{2} I_{Load} . \]  \hspace{1cm} (6.4)

From Fig. 6.1, the inverter current \( I_i \) can be expressed as the combination of the inductor current \( I_L \) and the diode current \( I_d \):

\[ I_i = I_{L1} - I_{c1} = I_{L1} - I_{c2} = I_{L1} - (I_d - I_{L1}) = 2I_L - I_d , \]  \hspace{1cm} (6.5)

where \( I_{c1} = I_{c2} \), \( I_{L1} = I_{L2} = I_L \).

The current through the diode cannot be lower than zero; therefore, the maximum available current for \( I_i \) is \( 2I_L \). However, in the following condition, the load peak current can be higher than \( 2I_L \):

\[ \sqrt{2} I_{Load} > \frac{3M \cdot I_{Load} \cdot \cos \phi}{\sqrt{2}} . \]  \hspace{1cm} (6.6)

This means that when the product of the modulation index and the load power factor is lower than \( \frac{2}{3} \), the inverter might have some new operation modes. Figure 6.2 shows some new operation modes that might occur under this condition.

Fig. 6.2. New operation modes during self-boost.
During the traditional zero state, the current to the inverter, \( I_o \), is zero. When the inverter turns into an active state, and the current required to the inverter, \( I_i \), is higher than what is available—\( 2I_L \) when Eq. (6.6) is met—the free-wheeling diode \( D_{f1} \) and \( D_{f2} \) will be turned on to provide the required current to the load, which forms an undesired shoot-through state as shown in Fig. 6.2 (a). During the shoot-through state, the inductor current increases linearly, until

\[
2I_L = I_i. \tag{6.7}
\]

Then the inverter turns into the active state shown as in Fig. 6.2(b), in which the input end diode is still reverse biased, and the current to the load is provided by the capacitor only until the next switching action to turn the inverter into a zero state or another active state.

The unwanted shoot-through state will boost the output voltage; this could be a good function to start up the fuel cell, especially for cold start. However, this could be a problem for other applications and needs to be controllable.

### 6.2 Control of Self-Boost

The self-boost phenomenon can be controlled by using a battery in the system, which is necessary for an FCV anyway.

The self-boost is caused by the low inductor current. In a system with a battery, the inductor current can be controlled independently by charging or discharging the battery. One possible configuration of the Z-source inverter for FCVs with a battery is shown in Fig. 6.3.

![Fig. 6.3. Configuration of the Z-source inverter with a high-voltage battery.](image)

With the configuration shown in Fig. 6.3, we can easily obtain the following equations.

\[
I_d = I_c + I_{L1} = I_c + I_d - I_{L2} \Rightarrow I_i + I_d = I_{L1} + I_{L2}, \tag{6.8}
\]
\[
\bar{I}_f = \bar{I}_{L1}, \quad \text{(6.9)}
\]
\[
\bar{I}_{L2} = \bar{I}_i, \quad \text{(6.10)}
\]
where \(\bar{I}_f\) is the average fuel cell current and \(\bar{I}_{L1}\) is the average inductor current through \(L_1\).

From these equations, we need to maintain the sum of the two inductor currents, \(I_{L1}\) and \(I_{L2}\), always higher than the load peak current, to avoid self-boost. From Eq. (6.10), the current through \(I_{L2}\) in the worst case is zero when the power factor is zero. Therefore, keeping \(I_{L1}\) greater than the peak load current would be more than enough.

The fuel cell voltage is determined by its output current; on the other hand, the fuel cell current can also be calculated by its output voltage. For a fuel cell with the VI curve, shown in Fig. 6.4, the voltage and the current has a one-to-one relationship, therefore, for any given fuel cell voltage, the fuel cell current is determined.

\[
\begin{align*}
& \text{Fig. 6.4. Fuel cell VI curve.} \\
& \text{In the fuel cell-battery system shown in Fig. 6.3, assume the battery voltage stays fairly constant at } V_b. \text{ The inductor current ripple is} \\
& \Delta I_{L1} = T_0 \frac{V_b}{L}, \quad \text{(6.11)} \\
& \text{where } T_0 \text{ is the shoot-through time. To make sure that self-boost will not occur, the minimum inductor current has to be higher than the peak load current; thus the target average inductor current is} \\
& I_{L1,\text{target}} = \sqrt{2} I_{\text{Load}} + \frac{V_b}{2L} T_0. \quad \text{(6.12)}
\end{align*}
\]
Based on our previous analysis, the fuel cell average current should equal the inductor average current, \( I_{L1\text{target}} \); and as seen in Fig. 6.4, the resulting target voltage is \( V_{\text{target}} \). The voltage relationship between the fuel cell voltage and the battery voltage satisfies Eq. (6.13):

\[
V_b = \frac{1-T_b \cdot V_f}{1-2T_0}.
\]  

(6.13)

With the known value of the battery voltage and the desired value of the fuel cell voltage, \( V_{\text{target}} \), the shoot-through duty ratio can be calculated.

For a given battery voltage, \( V_b \), the fuel cell voltage cannot be higher than \( V_b \); thus the minimum average current (\( I_{\text{min}} \)) through \( L_1 \) is the corresponding current of the fuel cell with voltage of \( V_b \). Therefore, when the peak load current is lower than \( I_{\text{min}} \), self-boost will not happen at all.

**6.3 Simulation Verifications**

To verify this method, a simulation was implemented. The inverter system is shown in Fig. 6.3; the fuel cell has the characteristics shown in Fig. 6.4. The battery in the system is 320 V. Figure 6.5. shows the simulation results. The load power factor for both cases in the simulation is 0.3, and the modulation index is both 0.1. For the case shown in (a), the load current is less than 50 A, which is less than the fuel cell current at 320 V. Therefore, no shoot-through is needed to control the fuel cell voltage. Based on the results shown in (a), the IPM PN voltage is a straight line, meaning that no unwanted shoot-through exists and therefore no self-boost occurs. For the case shown in (b), the load peak current is 200 A, which is higher than the fuel cell voltage at 320 V. Thus shoot-through is introduced to control the fuel cell voltage. The fuel cell voltage is reduced to 250 V, which makes the output current of the fuel cell 200 A. Together with the current through \( L_2 \), the sum of the two inductor currents becomes higher than the load peak current. Therefore, no unwanted shoot-through occurs.
6.4 Simulation and Experimental Verification of Handling Load Dynamics

With a battery in the system, as shown in Fig. 6.3, the inverter is able to handle load dynamics and regenerative braking power. The detailed analysis, simulation, and experimental verification can be found in ref. [7].

6.5 Summary

In this chapter, the self-boost phenomenon of the Z-source inverter without a battery is analyzed. This phenomenon is actually helpful for fuel cell cold start; however, it could be a problem for other applications. The detailed analysis and a discussion of the control method with a battery are provided. For an FCV system, a battery is always necessary; therefore, self-boost is completely controllable in an FCV system.
7. Summary

This report has summarized the results and findings of the work conducted on the Z-source inverter for FCVs. The major work completed in this project includes (1) development of a low-cost, high-efficiency traction drive Z-source inverter for FCVs; (2) a detailed comparison of the Z-source inverter with traditional inverter topologies; (3) the design and fabrication of a 30-kW Z-source inverter; and (4) testing and demonstration of the Z-source inverter developed.

First, a unique Z-source inverter was developed for FCVs. This inverter is especially suited for a fuel cell that requires unidirectional current/power flow and has a wide voltage range. At present, there are two existing topologies for FCVs: the traditional PWM inverter and the dc/dc boosted PWM inverter. The traditional PWM inverter topology uses the fuel cell voltage directly, which results in a limited CPSR for the motor and a freeze-start problem for the fuel cell. The dc/dc boosted PWM inverter topology suffers from high cost, low converter efficiency, and complexity. The specially developed Z-source inverter can solve these problems.

Second, a comprehensive comparison was conducted of the Z-source inverter and the two existing inverter topologies mentioned. The comparison results showed that the Z-source inverter is highly suitable for FCVs, having a lower cost (20% lower semiconductor ratings), higher conversion efficiency (1% higher), a wider CPSR (1.68 times that of the traditional PWM inverter), and greater reliability (no need for dead time).

A 30-kW Z-source inverter was designed and fabricated. The detailed design process included circuit design, 3-D design, and thermal design. To reduce the switching loss of the inverter, a new PWM scheme for shoot-through was introduced.

Finally, lab testing was conducted at Michigan State University and ORNL separately. Testing results confirmed the theoretical analysis and comparison results obtained and showed that the inverter had higher efficiency (up to more than 98%) and low harmonic distortion (i.e., low current and torque ripples).

A self-boost phenomenon of the Z-source inverter when the traction motor was operating at low speed, low power, and low power factor was observed during the test. This self-boost function was initially deemed a problem; however, it turned out to be a needed function for successful fuel cell startup, especially for freeze-starts. In addition, this self-boost can be totally controllable when a battery is incorporated into the inverter system. A detailed analysis of the
self-boost phenomenon and a control method using a battery was provided and confirmed by simulation.

In summary, the results of the project have shown high feasibility and many unique features of Z-source inverter for FCVs. As the automotive industry has recently been urged to develop ICE- and battery-based HEVs as the bridging technology to FCVs, this report addressed issues and feasibility for the use of the Z-source inverter in ICE HEV traction drive systems.

Future work may include

1. Including a battery in the system. The current system does not have a battery and thus cannot handle load dynamics. Including a battery and four-quadrant operation control of the inverter will provide a complete drive system for FCVs.
2. Developing a new bi-directional Z-source inverter for an ICE-battery hybrid drive system.
References


Appendices

A. Derivation of Key Equations

Equations (2.1) and (2.2)

The output phase rms voltage is

\[ V_p = \frac{V_i}{2\sqrt{2}} M. \]

With motor power factor of \( \cos \phi \), the output line RMS current is

\[ I_p = \frac{P_o}{3\cos \phi V_p}. \]

Because the line current is evenly shared by two switches in a line cycle, the average current through each switch is:

\[ I_{av} = \frac{P_o}{3\cos \phi V_i/(2\sqrt{2})M} \cdot \frac{\sqrt{2}}{2\pi} \cdot \frac{4P_o}{3\cos \phi V_i M} = \frac{8V_{max}P_o}{\cos \phi V_i M}. \]

The maximum voltage stress of the switches occurs when the output power is zero and the fuel cell voltage reaches its maximum value, which is

\[ V_s = V_{max}. \]

The total average switching device power of the circuit is

\[ (SDPR)_{av} = 6V_s \cdot I_{av} = \frac{8V_{max}P_o}{\cos \phi V_i M}. \]

The peak current through the switches is

\[ I_{pk} = \frac{P_o}{3\cos \phi V_i/(2\sqrt{2})M} \cdot \frac{\sqrt{2}}{2\pi} \cdot \frac{4P_o}{3\cos \phi V_i M} = \frac{8V_{max}P_o}{\cos \phi V_i M}. \]

The total peak switching device power of the converter is

\[ (SDPR)_{pk} = 6V_s \cdot I_{pk} = \frac{8V_{max}P_o}{\cos \phi V_i M}. \]

Equations (2.3) and (2.4)

For the switch in the boost converter, treating the switch and the diode as a switch cell, the maximum voltage it sustains is \( V_{DC} \), and the average current through it is

\[ I_{av} = \frac{P_o}{V_i}. \]

The average switching device power of the dc/dc converter is
Supposing the current through the inductor is constant, the peak current through the switch is the same as the average current. The peak switching device power is

\[(SDPR)_{DCpk} = \frac{P_o}{V_i} \cdot V_{DC} \cdot \]

The voltage stress of the inverter switches is \(V_{DC}\). The rms phase voltage at modulation index of \(M\) is

\[V_p = \frac{V_{DC}M}{2\sqrt{2}}.\]

The average current through switches under maximum power is

\[I_{avinv} = \frac{P_o}{3M \cos \phi V_{DC} / 2\sqrt{2}} \cdot \frac{2}{2\pi} = \frac{4P_o}{3 \cos \phi V_{DC} \pi M}.\]

The average switching device power of the system is

\[(SDPR)_{av} = 6 \cdot V_{DC} \cdot I_{avinv} + V_{DC} \cdot I_{av} = \frac{8P_o}{\cos \phi \pi M} + \frac{P_o}{V_i} \cdot V_{DC}.\]

The peak switch current of the inverter is

\[I_{pinv} = \frac{P_o}{3M \cos \phi V_{DC} / 2\sqrt{2}} \cdot \sqrt{2} = \frac{4P_o}{3 \cos \phi V_{DC} M}.\]

The peak switching device power of the system is

\[(SDPR)_{pk} = 6 \cdot V_{DC} \cdot I_{pinv} + V_{DC} \cdot I_{pk} = \frac{8P_o}{\cos \phi \pi M} + \frac{P_o}{V_i} \cdot V_{DC}.\]

Equations (2.5) and (2.6)

For the Z-source inverter, the current through the inverter switches consists of two elements, the current to the load and the current through them when the circuit is in the shoot-through state. Because of the symmetrical structure of the inverter, the current during shoot-through in terms of average is evenly distributed in three parallel paths. The average current value in shoot-through period through each switch is

\[I_{avss} = \frac{2}{3} I_L,\]

where \(I_L\) is the inductor current, which is
\[ I_L = \frac{P_o}{V_i} \]

While in active states, the average current is the same as in a conventional PWM inverter; therefore, the overall average current through inverter switches is

\[
I_{\text{avg}} = \frac{2}{3} I_L \frac{T_0}{T_s} + \frac{\sqrt{2} P_o}{3 V_o \cos \phi \pi} (1 - \frac{T_0}{T_s}) = \frac{2}{3} I_L \frac{T_0}{T_s} + \frac{4(\sqrt{3} M - 1) P_o}{3 V_o \cos \phi \pi M} (1 - \frac{T_0}{T_s})
\]

where \( T_0 \) is the shoot-through period in a switching cycle \( T_s \), \( V_o \) is the rms output phase voltage. \( T_0 \) and \( V_o \) can be expressed as

\[
T_0 = \left(1 - \frac{\sqrt{3}}{2} M\right) T_s
\]

\[
V_o = \frac{M}{\sqrt{3} M - 1 2\sqrt{2}} V_i
\]

Voltage stress of the inverter switches is

\[
V_s = B V_i = \frac{V_i}{\sqrt{3} M - 1}
\]

The average switching device power of the inverter is

\[
(SDPR)_{\text{av}} = 6 I_{\text{avg}} V_s = 4 I_L V_i \frac{T_0}{T_s (\sqrt{3} M - 1)} + 8 \frac{P_o}{\cos \phi \pi M} \left(1 - \frac{T_0}{T_s}\right) = \frac{2 P_o (2 - \sqrt{3} M)}{(\sqrt{3} M - 1)} + \frac{4\sqrt{3} P_o}{\cos \phi \pi}
\]

The peak current through the switches occurs during shoot-through. To calculate the peak current through the switches, we suppose that when the switches are on, they are pure resistors with the same resistance, which is shown in Fig. A.1.

![Fig. A.1 Inverter model during shoot-through.](image)

Based on this model, we can have the following equations
\[ I_{s2} + I_{s1} = \frac{4}{3} I_L, \]
\[ I_{s1} - I_{s2} = I_s, \]
from which we can get
\[ I_{s1} = \frac{1}{2} I_s + \frac{2}{3} I_L. \]

The peak current through the switch S1 occurs when the line current of phase A is at its peak. The peak switching device power of the inverter is
\[
\left( SDPR \right)_{pk} = 6 I_{pk} V_s = 6 \frac{V_i}{\sqrt{3} M - 1} \left( \frac{2 P_o}{3 V_i} + \frac{4 P_o}{2 \cos \varphi M V_i} \right) = \frac{4 P_o}{\sqrt{3} M - 1} + \frac{4 P_o}{\cos \varphi M}. 
\]

Equation (2.7)

Because of the internal impedance of the fuel cell, it outputs constant dc current. The peak voltage ripple of the capacitor occurs at maximum power, when the power factor of the motor is pretty high and there is no current fed back to the capacitor from the inverter. In a line cycle, the maximum voltage ripple across the capacitor occurs when the longest zero state happens. The maximum interval of open circuit in one cycle is
\[
T_o = T_s \times \max(1 - (M \sin(\alpha) - M \sin(\alpha - \frac{2}{3} \pi))) = \frac{\pi}{6} = (1 - \frac{3}{4} M) T_s.
\]

During this interval, the current through the capacitor is the current from the fuel cell, which is
\[ I_f = \frac{P_o}{V_i}. \]

Therefore, the maximum voltage ripple across the capacitor is
\[
\Delta V_c = \frac{T_0 I_{DC}}{C} = \frac{P_o T_s}{V_i C} (1 - \frac{3}{4} M).
\]

Equation (2.8)

Similar to the conventional PWM inverter, the maximum voltage ripple occurs at maximum power and maximum zero states. Suppose the dc/dc converter and the inverter share the same
carrier. In the period when the switch in the dc/dc converter is off, the current flow into the capacitor from the boost converter is

\[ I_{c1} = \frac{P_o}{V_i} \]

In the active state at this instant, the current flowing into the inverter is

\[ I_{c2} = \cos \phi I_{pk} = \cos \phi \frac{4P_o}{3V_{DC}M}. \]

During the period when the switch in the dc/dc converter is off, there is no current from the capacitor to the inverter for \((1 - \frac{3}{4}M)T_s\), and the amount of current \(I_{c2}\) flows to the inverter for the rest of the time. Therefore, the voltage ripple across the capacitor is

\[ \Delta V_c = (I_{c2}(1 - D) - I_{c2}(\frac{3}{4}M - D)) \frac{T_s}{C} = \frac{P_o T_s}{V_i C} (1 - D) - \cos \phi \frac{4P_o T_s}{3V_{DC}M} (\frac{3}{4}M - D). \]

*Equations (2.10) and (2.11)*

During the shoot-through interval, the capacitor charges the inductor and gives out current. The voltage ripple across the capacitors can be estimated

\[ \Delta V_c = \frac{I_s T_o}{C}, \]

where

\[ T_o = \left(1 - \frac{\sqrt{3}}{2}M\right)T_s, \]

and \(I_s = \frac{P_o}{V_i}\).

We have

\[ \Delta V_c = \frac{P_o}{V_i} \left(1 - \frac{\sqrt{3}}{2}M\right) T_s \]

\[ \frac{C}{L} \]

When the inverter is in a shoot-through state, the voltage across the inductor is the voltage across the capacitor. Therefore the current ripple of the inductor can be calculated as

\[ \Delta I = \frac{V}{L} T_o, \]
where $V_c$ is the voltage across the $C$

$$V_c = \frac{V_i}{2} (1 + B) = \frac{V_i}{2} \sqrt{\frac{3M}{3M - 1}}.$$

We have

$$\Delta I = \frac{V_i \sqrt{3M}}{2L(\sqrt{3M - 1})} \left(1 - \frac{\sqrt{3}}{2} M\right) T_s.$$
B. MATLAB Files to Calculate Current Ripple Through Capacitors

B.1. Program for current ripple of conventional PWM inverter

```matlab
% The objective of this program is to calculate the RMS current ripple
% through the DC link capacitor of the traditional PWM inverter.
%
%M: Modulation index
%Vdc: DC link voltage
%Ipeak: Peak line current
%PF: power factor of the motor or the inverter load
%
The output result is a figure, which denotes the RMS current versus time
in a period of (pi/6----pi/2). The output value in the command widow is
the overall RMS current through the capacitor.

% parameter section

M=1;
Ipeakp=297;
PF=0.9;
Vdc=250;
alpha=pi/6:0.01:pi/2;
Vpeakp=M/2*Vdc;
fai=acos(PF);
va=Vpeakp.*sin(alpha);
vb=Vpeakp.*sin(alpha-2*pi/3);
vc=Vpeakp.*sin(alpha+2*pi/3);
lakep.*sin(alpha-fai);
lb=Ipeakp.*sin(alpha-fai-2*pi/3);
lc=Ipeakp.*sin(alpha-fai+2*pi/3);
vs=Vdc;
```
I_{av} = \frac{3}{2} \cdot V_{peak} \cdot I_{peak} \cdot \cos(f_{ai}) / V_{dc};

I_{ripple} = \sqrt{I_{av}^2 \cdot (1 - v_a/v_s + v_b/v_s) + (v_a-v_c)/v_s \cdot \left( I_a - I_{av} \right)^2 + (v_c-v_b)/v_s \cdot \left( I_b + I_{av} \right)^2};

I_r = I_{av}^2 \cdot (1 - v_a/v_s + v_b/v_s) + (v_a-v_c)/v_s \cdot \left( I_a - I_{av} \right)^2 + (v_c-v_b)/v_s \cdot \left( I_b + I_{av} \right)^2;

\text{plot}(\alpha, I_{ripple})

\text{for } k = 1:1:105

\quad \text{mm}(k) = 1;

\text{end}

\text{IRMS} = \sqrt{I_r \cdot \text{mm}' / 105}
B.2. Program for current ripple of dc/dc boost+ PWM inverter

% The objective of this program is to calculate the RMS current ripple
% through the DC link capacitor of the traditional PWM inverter plus boost dc/dc converter.
% Please specify the following parameters:
% M: Modulation index
% Vdc: DC link voltage
% Ipeak: Peak line current
% PF: Power factor of the motor or the inverter load
% D: Duty cycle of the dc/dc converter
% The output result is a figure, which denotes the RMS current versus time
% in a period of (pi/6--pi/2). The output value in the command widow is
% the overall RMS current through the capacitor.

M=1
Ipeakp=176;
PF=0.9;
Vdc=420;
D=0.4;
n=0;
fai=acos(PF);
Vpeakp=M/2*Vdc;
for alpha=pi/6:0.0436:pi/2;
n=n+1;
a=Vpeakp.*sin(alpha);
b=Vpeakp.*sin(alpha-2*pi/3);
c=Vpeakp.*sin(alpha+2*pi/3);
a=Ipeakp.*sin(alpha-fai);
b=Ipeakp.*sin(alpha-fai-2*pi/3);
\[ I_c = I_{peakp} \cdot \sin(\alpha - \phi + 2\pi/3); \]
\[ v_s = V_{dc}; \]
\[ I_{av} = 3/2 \cdot V_{peakp} \cdot I_{peakp} \cdot \cos(\phi) / V_{dc}; \]
\[ I_{boost} = I_{av} / (1 - D); \]
\[ \text{if } D \leq (v_b + V_{dc}/2) / V_{dc} \]

\[ I_{ri}(n) = \sqrt{(V_{dc}/2 - v_a) / V_{dc} \cdot I_{boost}^2 + (v_a - v_b) \cdot (I_b + I_{boost})^2 + ((v_b + V_{dc}/2) / V_{dc} - D) \cdot I_{boost}^2}; \]
\[ m(n) = 1; \]
\[ \text{elseif } D \leq (v_c + V_{dc}/2) / V_{dc} \]

\[ I_{ri}(n) = \sqrt{(v_a - v_c) / V_{dc} \cdot (I_a - I_{boost})^2 + (v_b - v_c) / V_{dc} \cdot I_{boost}^2 + ((v_c + V_{dc}/2) / V_{dc} - D) \cdot (I_b + I_{boost})^2 + (v_b + V_{dc}/2) / V_{dc} \cdot (I_b)^2}; \]
\[ m(n) = 2; \]
\[ \text{elseif } D \leq (v_a + V_{dc}/2) / V_{dc} \]

\[ I_{ri}(n) = \sqrt{(v_a + V_{dc}/2) / V_{dc} - D) \cdot (I_a - I_{boost})^2 + (v_a - v_c) / V_{dc} \cdot (I_a)^2 + (v_b - v_c) / V_{dc} \cdot (I_b)^2 + (v_b - v_a) / V_{dc} \cdot I_{boost}^2}; \]
\[ m(n) = 3; \]
\[ \text{else } I_{ri}(n) = \sqrt{(1 - D) \cdot I_{boost}^2 + (v_a - v_c) / V_{dc} \cdot (I_a)^2 + (v_b - v_c) / V_{dc} \cdot (I_b)^2}; \]
\[ m(n) = 4; \]
\[ \text{end} \]
\[ \text{end} \]

\[ \alpha = \pi / 6:0.0436:\pi / 2; \]
\[ \text{plot}(\alpha, I_{ri}) \]
\[ \text{hold} \]
\[ \text{for } k = 1:1:25 \]
\[ \text{mm}(k) = 1; \]
\[ \text{end} \]
\[ \text{IRMS} = \sqrt{((I_{ri})^2 \cdot \text{mm}^2 / 25)} \]
B.3. Program for current ripple of Z-source inverter

% The objective of this program is to calculate the RMS current ripple
% through the DC link capacitor of the traditional PWM inverter plus boost dc/dc converter.
% Please specify the following parameters:
% M: Modulation index
% Vi: input voltage
% Ipeak: Peak line current
% PF: power factor of the motor or the inverter load
%
% The output result is a figure, which denotes the RMS current versus time
% in a period of (pi/6----pi/2). The output value in the command widow is
% the overall RMS current through the capacitor.

Ipeakp=185.7;
PF=0.9;
vin=250;
M=0.9
fai=acos(PF);
alpha=pi/6:0.01:pi/2;
va=M.*sin(alpha)+M/6.*sin(3.*alpha);
vb=M.*sin(alpha-2*pi/3)+M/6.*sin(3.*alpha);
vc=M.*sin(alpha+2*pi/3)+M/6.*sin(3.*alpha);
la=Ipeakp.*sin(alpha-fai);
lb=Ipeakp.*sin(alpha-fai-2*pi/3);
lc=Ipeakp.*sin(alpha-fai+2*pi/3);
vs=2;
Vpeakp=M/(sqrt(3)*M-1)*vin/2;
Iav=3/2*Vpeakp*Ipeakp*cos(fai)/vin;
Iripple=sqrt(Iav^2.*(1-va/2+vb/2)+(va-vc)/vs.*(Ia-Iav).^2+(vc-vb)/vs.*(Ib+Iav).^2);
plot(alpha,Iripple)
for k=1:1:105
    mm(k)=1;
end
IRMS=sqrt(Iripple.^2*mm'/105)
C. Detailed Gate Drive Schematic
D. Related Publications


