Optimal Control of Transient Behavior in Coupled Solid State Lasers

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Abstract

We applied the optimal control theory to substantially reduce transition times between in-phase and out-of-phase states in coupled solid state lasers. The control is a time-varying optical field that is injected into the cavities of each laser. Numerical simulations indicate that the transient times can be reduced by an order of magnitude when increasing briefly the injection strength by a factor of three.

Laser arrays are promising for space communication applications. Such applications require compact sources with high optical intensities and fast switching times \cite{1}. Recently, both solid state and semiconductor laser arrays have been investigated, and various relevant aspects of their dynamical behavior such as chaotic synchronization, chaotic communication, and amplitude drop-out have already been reported.

The most efficient mode of operation of the array is realized when its elements are synchronized to an in-phase (IP) state, such that the output interferes constructively and the light intensity at the central lobe scales as $N^2$, where $N$ is the number of lasers in the array. Unfortunately, the IP state is unstable; in general, the system is driven to the stable out-of-phase (OP) state, whose destructive interference pattern results in low output intensities at the central lobe. The IP behavior can be stabilized by injecting a common driving field into the laser array elements. For sufficiently high driving amplitude, the elements are entrained and the output intensities interfere constructively. Full entrainment of the array is realized above a certain threshold, determined by the coupling of the array elements.

Besides synchronization, an equally important issue in applications is the time required to reach the IP behavior from an arbitrary state. In particular, it is desirable to minimize transient times between IP and OP states (upon removal of the injected entrainment field). This aspect is important, for example, in fast switching and communications applications.

We apply optimal control (OC) to reduce the transient switching times between OP and IP states in an array of two coupled solid state lasers as described by the phase model, which has been shown to capture the main features of the full dynamics of the array for a broad range of parameters \cite{2}:

\[
\begin{align*}
\dot{\delta}_1 &= \delta_1 + \kappa \sin(\phi_2 - \phi_1) - A_e \sin \phi_1, \\
\dot{\phi}_1 &= \delta_2 + \kappa \sin(\phi_1 - \phi_2) - A_e \sin \phi_2.
\end{align*}
\]

In the system above, $\delta_i$, $i=1,2$ are the detunings, $\kappa$ is the coupling constant, and $A_e$ is the amplitude of the injected field.

To our knowledge, no control technique - optimal or otherwise - has been applied to reduce transient times in arrays of lasers. We target the approach at a system of coupled solid state lasers and demonstrate its efficiency. The OC method \cite{3} is completely general and systematic and does not depend essentially on internal features of the system, such as characteristic times. Since it tailors precisely the effort to the desired task, the OC method keeps the cost at its possible minimum and yields significant reductions of the transient time by a factor of ten, without resulting in overshoots.
In Figure 1, we present the results when initially the two lasers are in the OP state. The value of the injection amplitude $A_0$ to obtain the IP behavior is given by $A_0 = 4|\kappa| = 4$, since in our simulations $\kappa = -1$. We use the OC to decrease the transient time to reach the IP behavior. We have chosen seven upper bounds between 4 and 30. The corresponding time to reach the 95% (blue top line), 85% (green middle line), and 75% (red bottom line) of the total output intensities for different OC values. As expected, the transient time shortens as controls are allowed to take higher values.

Similar results are obtained for the transition between IP to OP states. Our results demonstrate that OC is a very efficient tool for significantly reducing the transition times between the IP and OP states for the array of coupled lasers.

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**References**