Fission Multiplicity Detection with Temporal Gamma-Neutron Discrimination from Higher Order Time Correlation Statistics

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by

Richard B. Oberer

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Contents

1 Introduction

The subject of this thesis is the nondestructive assay (NDA) of nuclear materials.[1] NDA is made possible by the fact that fissile and fertile transuranic isotopes emit characteristic radiations. Fission Multiplicity Detection (FMD) is the name of the standard technique used in NDA of nuclear materials in the 1960s and 1970s. It was characterized by the use of fast plastic scintillating detectors. These systems were used in both active and passive mode. These FMD systems were eventually replaced by thermal well counters as the standard NDA technique. The thermal well counters use ^{3}He detectors embedded in a moderator. Among the passive neutron assay techniques, neutron multiplicity counting (NMC) in a thermal well counter is the preferred technique for the determination of fissile mass when spontaneous fission yields are significant. It is used in conjunction with gamma-ray spectroscopy to determine the isotopic composition of a sample.

The major problem with the use of fast plastic detectors as used in FMD is that both neutrons and gamma rays are detected. The pulses from the two are indistinguishable. The disadvantages of this indistinguishability between neutron and gamma rays is explained further in the explanation of NMIS multiplicity in Section 2.5.

[1] See reference [21] and [22] for a good overview of NDA

In the case of LANL's Random Driver, the detectors were shielded with 5 cm of lead to suppress gamma detections. In addition the gamma-gamma peak in the detector-detector cross correlation was blocked. In a comparison with the active well, the major disadvantage of the Random Driver was it's bulk.[17] It weighed a metric ton which was ten times heavier than active well. An advantage of the fast plastic detectors is the very fast response time. Because of the long detector die away of the thermal well counters, the time bins are 250 ns wide and the window 3000 ns long. For a fast plastic detector, a window of 50 ns is sufficient. This shorter time window will result in a factor of 60 reduction in background and accidental correlations.

For this thesis, a technique will be developed to use higher-order time correlation statistics to distinguish combinations of neutron and gamma ray detections in fast plastic detectors. NMIS will be used as the time analyzer. A system of analysis to describe these correlations will be developed based on simple physical principles. Other sources of correlations from non-fission events will be identified and integrated into the analysis developed for fission events. A number of ratios and metrics will be identified to determine physical properties of the source from the correlations. A number of these ratios are set out in Section 5.3.

As a demonstration of this technique, a ^{252}Cf source which is well known, will be measured using fast plastic scintillating detectors using the temporal discrimination technique and analysis. The technique will first use the temporal resolving characteristics of an instrumented ^{252}Cf source to measure the source. Although this technique is not useful as an NDA technique, it is useful in the development of the theory of the temporal technique. The technique will then be extended to a non-instrumented source. The same ^{252}Cf source will be measured with this technique in the non-instrumented configuration to demonstrate the relation between the two configurations. Various data about the source will be extracted and analyzed from the time correlation statistics. To account for detector dead-time, an alternative analytical technique will also be developed.

The current and previous techniques of passive neutron NDA will first be reviewed in Section 2 of this proposal. The mathematics necessary to represent the time correlations will then be reviewed in Section 4. An analytical representation for the correlations between the source and a detector, between two detectors, between the source and two detectors, and finally between three detectors resulting from a fission event will be developed in Section 5 of this proposal. From this analysis, the temporal separation of the gamma and neutron combinations becomes apparent. Various ratios of these correlations will be presented will be proposed in Section 5.3.

2 Current and past practices of passive neutron multiplicity counting techniques for NDA

This section describes the current practice of passive NMC for NDA with thermal well counters including the more recent innovation of Time Interval Analysis (TIA). Fission multiplicity detection (FMD) will then be briefly described along with the previous NMIS implementation of multiplicity.

In NMC it is the quantity of an unknown fissile mass that is of interest. In addition to this unknown, the chemical composition, mixture and environment of the mass may also be unknown. Because of these other unknowns, neutrons may be generated from (α, n) reactions rather than fission. In addition, the neutron detection efficiency may not be well known. The techniques used to overcome these difficulties will be described.

2.1 Detectors

The nature of the detectors used in the current practice of passive neutron multiplicity counting for NDA must be contrasted with the fast recoil detectors used in FMD and this research. The detectors used in NMC are thermal neutron well counters. These well counters are comprised of ^3He tubes embedded in an annular polyethylene moderator. The sample being measured is placed in the well of the annulus which is lined with cadmium to decouple the sample from the moderator. ^3He detectors operate through the $^3He(n,p)^3H$ thermal capture reaction. As a consequence, these detectors do not detect gamma rays. Furthermore, once a neutron is detected it is removed from the system.

The fast recoil detectors operate on proton recoil for neutron detection. Gamma rays are detected as well as neutrons, and a detection of a neutron or gamma ray does not remove it from the system. The neutron detection efficiency for the well counters can be as high as 50% or more. In contrast, the efficiency of the 4 inch detectors used in this research is less than 1% when the solid angle is considered. The detection efficiency of the well counter is greatest at about 1 MeV. Relatively high detection efficiency extends down to several keV or lower. The fast plastic scintillators are blind below the threshold setting which is typically set at 1 MeV in this research.

The time response of the detectors is also entirely different. The time response of the well counters is characterized by a single exponential die-away with a time constant on the order of 50 μs. The time response of the fast plastic detectors is dominated by the time of flight of the radiation. For the threshold neutrons with a velocity of 1.4 cm/ns, this time is 17 ns at a distance of 23 cm.

2.2 Neutron Multiplicity Counting (NMC)

It is the total mass of plutonium that is determined from a passive neutron multiplicity measurement. The isotopic composition of the plutonium is determined by another method, usually gamma spectroscopy.

If the plutonium is in oxide form, the neutron yield from (α, n) reactions can be considerable. Neutron coincidence counting was developed to measure the Pu mass when the (α, n) reaction rate was unknown. Neutrons from the (α, n) reaction are always generated in singles. Neutrons from spontaneous fission, on the other hand, are generated in multiples ν, with a probability density of $p(\nu)$. Coincident counting is essentially identical to determining the doubles rate in neutron multiplicity counting.[2]

In addition, the detection efficiency ϵ_n or sample multiplication may also be unknown. One of these additional unknown can be determined if, in addition to the doubles rate in

[2] A more comprehensive introduction to neutron multiplicity counting can be found in reference [9]. In addition, reference [1] provides insight and historical perspective.

coincident counting, the triples rate is also determined. This extension is in essence multiplicity counting. Both the doubles and triples rate are determined from shift register statistics.

A typical shift register might have 128 bits and operate on a 4 MHz clock. Each bit would therefore represent 0.25 μs. The gate window W would be 32 μs which is the total time required for a bit to travel through the shift register. A pulse from the detector sets the first bit in the shift register. This bit is shifted one bit at a time down the register every 0.25 μs. A counter adds one for each bit that enters the shift register and subtracts one for each bit that exits the shift register 32 μs later. This counter therefore contains the number of pulses in the shift register at any time. This number represents the number of detected pulses which occurred in the previous 32 μs.

The content of the counter must be tallied at various times. Tallying the shift register is often referred to as triggering the gate. The content of the shift register is tallied when each pulse exits the shift register. The result is the number of pulses following every pulse within the gate window. Typically the tally is triggered after a short delay of 3 to 4.5 μs called a pre-delay. A gate triggered in this manner is called the R+A or reals plus accidentals gate.

The accidental coincidences are tallied with the A gate. It is triggered after a long but constant delay after the R+A gate. The delay for the A gate is set sufficiently long to allow all correlated events to die away. A typical value for this delay might be 4096 μs.

Once the measurement is accomplished, there will be two distributions of shift register counts. One distribution is for the R+A gate, the other is for the A gate. The distributions contain tallies of the number of times that 0, 1, 2, ... n bits were set in the shift register at the time it was triggered. Properly normalized by the total counts, we can call these distributions $p(n)$ from the R+A gate and $q(n)$ for the A gate. These distributions represent the probability of finding 0, 1, 2, ... counts in the shift register at the appropriate trigger times. From these distributions, the singles, doubles, and triplets are computed.

The distribution that is desired is the probability that n correlated neutrons are in the shift register. This probability is usually called r(n). The probability of finding no counts in the R+A gate is $p(0) = r(0)q(0)$. That is, the probability of finding no correlated counts and no accidental counts. One count in the R+A shift register can come from one correlated count and no accidentals or from one accidental and no correlated counts. The probability of this occurring is therefore $p(1) = r(0)q(1) + r(1)q(0)$. the probability of two counts is $p(2) = r(0)q(2) + r(1)q(1) + r(2)q(0)$ and so on. The singles, doubles and triples are then computed from the moments of the r(n) distribution.[14]

The singles, doublets and triplets can then be related to the actual physical parameters of the sample by the neutron multiplicity point equations.[11]

Usually three of the variables, fission rate N, detector efficiency ϵ_n, multiplication M, or α are solved from the singles doubles and tripples. The effective mass of ^{240}Pu is then calculated from the fission rate N and the specific fission rate of ^{240}Pu.

2.3 Time Interval Analysis (TIA)

A new approach to neutron multiplicity analysis has been recently proposed by Baeten.[1, 2, 3, 4, 5, 6] It departs from the traditional shift register approach in favor of time interval analysis. In this sense it is similar to the time correlations produced by NMIS. In terms of NMIS signatures, it is based on the detector autocorrelation for doublets and a detector auto-bicorrelation for triplets. It differs from NMIS in that the detector head is that used in the traditional multiplicity techniques as described above.

Although TIA is a new approach to multiplicity analysis, virtually identical equations were derived by Hansen in 1968.[12] Baeten uses the same definition of the delay variables, i. e., $\tau_1 = t_2 - t_1$ and $\tau_2 = t_3 - t_2$. An arguably more natural definition for τ_2 would refer it back to the first detection also, which would make $\tau_2 = t_3 - t_1$. One difference, however, is that Hansen casts his equations in terms of the Rossi-α and includes multiplication terms. Baeten's equations are in terms of the detector die away τ_D and does not include multiplication.

2.4 Fission Multiplicity Detectors (FMD)

Previous to the current practice of multiplicity counting fast recoil detectors were used in a technique called fission multiplicity detection (FMD).[3] These systems were popular in the 1960s and 1970s. Several of these systems were commercially available. One of the earliest of these was the Isotopic Source Assay System (ISAS) developed by Gulf General Atomics. It was used in passive mode for the assay of plutonium or with a neutron source for the assay of uranium. Another commercial system was the Isotopic Source Adjustable Fissionometer (ISAF) designed by IRT Corporation formerly Gulf Radiation Technology. It used an Am-Li source in active mode. There were two versions of the Random Driver. One was from Los Alamos Scientific Laboratory. It used an Am-Li source. The other was from National Nuclear Corporation and used a Pu-Li source in active mode.

These systems all used two or four fast plastic detectors sensitive to both fast neutrons and gamma rays. Coincidences were counted between detectors. Gamma rays were not explicitly treated in their analysis however. In the case of LANL's Random Driver, the detectors were shielded with 5 cm of lead to suppress gamma detections. In addition the gamma-gamma peak in the detector-detector cross correlation was blocked. In a comparison with the active well, the major disadvantage of the Random Driver was it's bulk[17]. It weighed a metric ton which was ten times heavier than active well.

2.5 NMIS Multiplicity

A kind of multiplicity capability has been incorporated into NMIS previously.[19, 24, 20] This section will describe this implementation. NMIS essentially mimics a traditional shift register, albeit capable of 1 ns time bins. If it were hooked up to a well counter, one would have an ordinary multiplicity system.

The fast plastic scintillating detectors used in NMIS make these multiplicities entirely different from traditional NMC. Unlike the He-3 detectors used in traditional multiplic-

[3]For a description of FMD see [22] pages 174 to 178 and [10] at paragraphs 8.6, 8.7 and 9.5.

ity, the plastic scintillators also detect gamma rays. So the multiplicity is a combined neutron and gamma multiplicity. In addition, the fast detectors are blind to neutrons below the threshold energy for neutrons usually set at 1 MeV. Neutrons are not removed from the system on detection. This can cause a single radiation to be counted more than once or to be reflected back into the sample. A more complete comparison between the detectors is described previously in Section 2.2.

Why not simply use a combined gamma-neutron multiplicity as the assay measurement like NMIS or FMD? First, there is nothing analogous to the point equations used in NMC to relate the combined multiplicity to effective mass, (α, n) rate, multiplication and detector efficiency. Second, although the fast plastic scintillators detect copious gamma rays, they are not especially well suited for that purpose. Because of the low Z of the detectors, the probability of interaction is low and furthermore the interactions that do occur are primarily Compton scattering interactions. So for a Compton scattering event, there is a substantial probability that the energy deposited will be below the threshold for gamma rays. Second, the gamma multiplicities are not well known. Third, gamma multiplicities have numerous ill-behaved characteristics which make consistent measurements difficult. Unlike neutrons, the gamma multiplicities are energy dependent. Therefore small changes in the threshold will change the multiplicity. Attenuation of the gamma rays, which is also energy dependent, will change the observed multiplicities. Many other events besides fission produce multiplicities of gamma rays. For example, the beta decay of the fission products typically emit a cascade of several gamma rays.

3 Experimental Details

The proposed experimental setup is shown in Figure 1. It is comprised of three plastic scintillating detectors arranged in an equilateral triangle around a ^{252}Cf source contained in an ionization chamber. The ^{252}Cf source is intended to represent the unknown sample to be assayed such as plutonium. In practice, the sample would not be contained in an ion-chamber. The purpose of the ion-chamber is to provide additional timing information for scientific inquiry. The source was developed by Mihalczo.[18, 13] It consists of a 1 cm diameter spot of ^{252}Cf plated onto the center of a circular platinum disk.

The output of the detector photomultiplier tubes goes into a constant fraction discriminatory (CFD). The discriminator signals the time of a detection event with a 0 to 1 logic transition. At this point, the signal is a continuous variable of time. The NMIS time-analyzer then effectively converts this continuous time signal into a discrete time variable by assigning it to a time bin of width Δt in a block of data with N time bins. Both Δt and the block size N are adjustable. Typically $\Delta t = 1ns$ and $N = 512$. The length of a block is then $512ns$ or a little over a half a microsecond. Compare this time to the Δt of $0.25\mu s$ for the typical shift-register. There are five input channels numbered 1 through 5.

The NMIS time-analyzer then computes and tallies the time between detections in all pairs and triplets of detectors. These are the correlations. They include blocks with themselves for the autocorrelations.

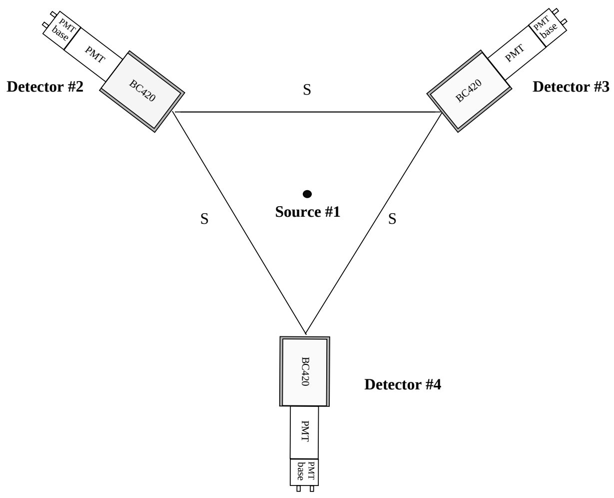

Figure 1: Experimental setup.

4 Analytical foundation

4.1 Definition of the correlation function

The correlation of the functions $x_1(t_1)$ and $x_2(t_2)$ of the random variables t_1 and t_2 for a stationary process is defined as

$$C_{12}(\tau) = E[x_1(t_1)x_2(t_2)] = E[x_1(t)x_2(t - \tau)]. \tag{1}$$

For the time intervals of interest, the NMIS signals can be treated as stationary. That is, the correlation function depends only on the time difference between the signals, $\tau = t_1 - t_2$. The random variables are the times t_1 and t_2 of detection events. The function x(t) is a tally of the number of detections in the interval t, $t+dt$. The expectation operator $E[x]$ can be a bit confusing. By definition it is

$$E[x_1(t_1)x_2(t_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 p(x_1, x_2, t_1, t_2) dx_1 dx_2 dt_1 dt_2. \tag{2}$$

Assuming that the random variables x_1 and x_2 are independent of t_1 and t_2, the joint probability density can be factored so that

$$\begin{aligned} E[x_1(t_1)x_2(t_2)] &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2, p(x_1, x_2) dx_1 dx_2 p(t_1, t_2) dt_1 dt_2 \\ &= \overline{x_{12}} p(t_1, t_2) dt_1 dt_2. \end{aligned} \tag{3}$$

The random variables x_1 and x_2 are counts, following binomial or Poisson probability distributions.

Focusing now in the time delay dependent probability distribution, it can be factored and a delay variable substituted

$$\begin{aligned} p(t_1, t_2)dt_1dt_2 &= p(t_1)p(t_2|t_1)dt_1dt_2 \\ \tau &= t_2 - t_1 \\ d\tau &= dt_2 \\ p(\tau)d\tau &= p(t_1)p(t_1 + \tau|t_1)dt_1d\tau \end{aligned} \tag{4}$$

Then integrating over the remaining time variable leaves $C_{12}(\tau)$ a function of the delay variable τ alone.

$$\begin{aligned} C_{12}(\tau) &= E[x_1(t)x_2(t-\tau)] \\ &= \overline{x_{12}} \int_{-\infty}^{\infty} p(t_1)p(t_1 + \tau|t_1)dt_1 \\ &= \overline{x_{12}}p(\tau) \end{aligned} \tag{5}$$

Equation (5) merely says that correlation can be separated into an average coincident count multiplied by a probability density of the delay variable τ. The next section will provide an example of how this definition is applied.

4.2 Correlation function in a fast neutron system

In this example the fast detectors used in this research are time correlated with a fission source. Assume an instrumented fission source has produced N_1 fissions during a measurement. Of these ϵ_f are detected in Channel 1. Each fission will produce $\overline{\gamma}$ gamma rays and $\overline{\nu}$ neutrons on average. The probability of detecting the gamma rays and neutrons is ϵ_g and ϵ_n respectively. The total number of fission-gamma and fission-neutron pairs will then be $\epsilon_f N_1 \overline{\gamma} \epsilon_g$ and $\epsilon_f N_1 \overline{\nu} \epsilon_n$ respectively.

The gamma rays will all arrive at the detector at nearly the same time. We can therefore represent the time distribution of this arrival time as

$$p_g(\tau_{12}) = \delta\left(\tau_{12} - \frac{d}{c}\right) \tag{6}$$

where d is the distance between the fission source and the detector, τ_{12} is the difference between the time of detection t_2 and the time of the fission at t_1 and c is the speed of light. The δ can be taken either literally or at least implying a narrow probability distribution.

The neutron fission spectrum is a function of energy, $\chi(E)$. The energy of a neutron is related to its velocity by

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{d}{\tau_{12}}\right)^2 \tag{7}$$

The relationship between the spectrum as a function of time and the spectrum as a function of energy is $\chi(\tau)d\tau = \chi(E)dE$ where $\frac{dE}{d\tau} = \frac{E}{\tau}$. Therefore the time distribution of neutron detections subsequent to a fission can be written

$$p_n(\tau_{12}) = \epsilon_n(\tau_{12})\chi(\tau_{12}) = \epsilon_n(\tau_{12})\chi\left(E \rightarrow \frac{1}{2}m\left(\frac{d}{\tau_{12}}\right)^2\right)\frac{E}{\tau_{12}}. \tag{8}$$

Substituting into Equation 5 results in

$$C_{12}(\tau_{12}) = \epsilon_f N_1 \overline{\gamma}\epsilon_g \delta\left(\tau_{12} - \frac{d}{c}\right) + \epsilon_f N_1 \overline{\nu}\epsilon_n(\tau_{12})\chi(\tau_{12}). \tag{9}$$

A measurement of C_{12} is shown in figure 2. Other correlations can be derived from this correlation as will be described in the next section.

4.3 Uncorrelated background and accidental correlations

Throughout most of this discussion it is assumed that the background and accidental correlations have been subtracted out. For bicorrelations,[4] the accidentals include an accidental correlation with a correlated pair in addition to the uncorrelated triplet $r_1 r_2 r_3$.

$$\begin{aligned} C_{123accidental}(\tau_{12}, \tau_{13}) =& r_1 C_{23}(\tau_3 - \tau_2) \\ &+ r_2 C_{13}(\tau_{13}) \\ &+ r_3 C_{12}(\tau_{12}) \\ &+ r_1 r_2 r_3 \end{aligned} \tag{10}$$

It should be understood that the doubles correlations C_{ij} in Equation 10 also have the background $r_i r_j$ subtracted out. In addition to accidental coincidences, uncorrelated background radiation are included in the accidentals in Equation 10.

5 Analytical expressions and temporal separation of neutron and gamma coincidences from fission

This section will first show the relationship between the source-detector correlation $C_{12}(\tau)$, the source-detector-detector bicorrelation $C_{123}(\tau_2, \tau_3)$, the detector-detector correlation $C_{23}(\tau_{23})$[5] and the three-detector bicorrelation $C_{234}(\tau_{23}, \tau_{24})$ from a spontaneous fission source.

[4] A more detailed treatment of the accidental coincidences and the relation to the covariance can be found in reference [15, 16]

[5] An alternative analytical treatment of these correlations can be found in Reference[8]

5.1 Mathematical expressions for various source and detector correlations

The first correlation to be considered is between the instrumented source and a detector separated by a distance d in air, i.e., there is no intervening material. A mathematical expression for this correlation was shown from Equation 9 to be

$$\begin{aligned} C_{12}(\tau) &= N_1 p(\tau) \\ &= N_1 \epsilon_f \left[\overline{\gamma} \epsilon_g \delta \left(\tau - \frac{d}{c} \right) + \overline{\nu} \epsilon_n(\tau) \chi(\tau) \right] \end{aligned} \tag{11}$$

All of the uncorrelated background has been removed from C_{12}. N_1 represents the number of fissions occurring in the instrumented source, and ϵ_f represents the probability of detecting the fissions in the instrumented source.

It is convenient in the development of the analysis to rewrite the correlation function as $C_{12}(\tau) = C_{fg}(\tau) + C_{fn}(\tau)$

where

$$C_{fg}(\tau) = N_1 \epsilon_f \overline{\gamma} \epsilon_g \delta \left(\tau - \frac{d}{c} \right) \qquad \text{represents the gamma response, and}$$

$$C_{fn}(\tau) = N_1 \epsilon_f \overline{\nu} \epsilon_n(\tau) \chi(\tau) \qquad \text{represents the neutron response.}$$

An actual measurement of this function is shown in Figure 2. The gamma response $C_{fg}(\tau)$ can clearly be seen at $\frac{d}{c} = 3.3ns$. It has a width of $\Delta_g \approx 10ns$. The neutron spectrum $C_{fn}(\tau)$ is temporally separated from the gamma peak. It extends from the time the fastest neutrons n_f are detected $\tau_f = \frac{d}{v_f}$ to the time that the threshold energy neutrons n_t are detected $\tau_t = \frac{d}{v_t}$. The threshold is typically $1MeV$ corresponding to neutrons which have a velocity of $1.4cm/ns$. The fast neutrons have a velocity of about $4.8cm/ns$. This velocity is observed from Figure 2. It ultimately comes from a combination of the Maxwellian distribution and detection efficiency.

The instrumented source can also be correlated with two detectors. This correlation is shown in Figure 3. The correlation function can be written as

$$C_{123}(\tau_2, \tau_3) = C_{fgg}(\tau_2, \tau_3) + C_{fgn}(\tau_2, \tau_3) + C_{fng}(\tau_2, \tau_3) + C_{fnn}(\tau_2, \tau_3) \tag{12}$$

where

$$C_{fgg}(\tau_2, \tau_3) = N_1 \epsilon_f \overline{\gamma(\gamma - 1)} \epsilon_g^2 \delta \left(\tau_2 - \frac{d_2}{c} \right) \delta \left(\tau_3 - \frac{d_3}{c} \right)$$

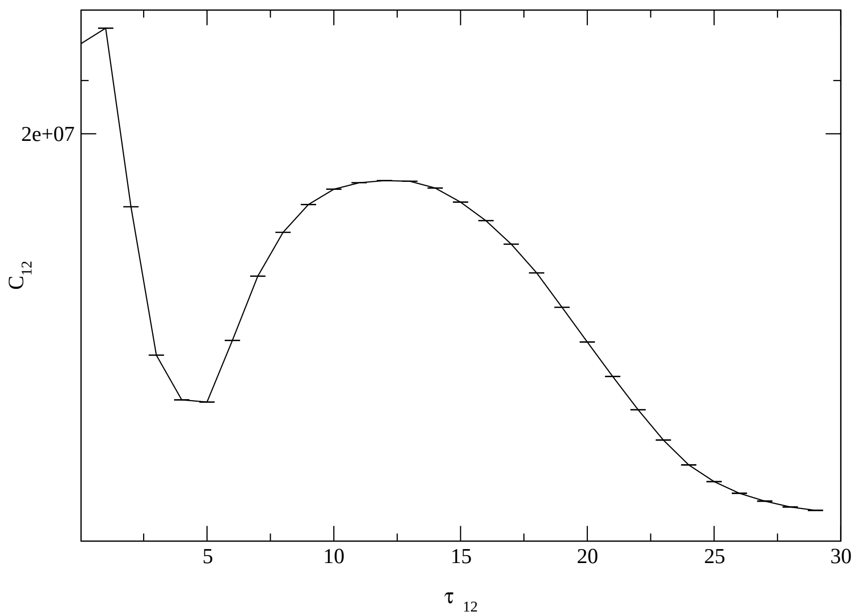

Figure 2: $C_{12}(\tau)$.

$$C_{fgn}(\tau_2,\tau_3) = N_1\epsilon_f\overline{\nu\gamma}\epsilon_g\delta\left(\tau_2 - \frac{d_2}{c}\right)\epsilon_n(\tau_3)\chi(\tau_3)$$

$$C_{fng}(\tau_2,\tau_3) = N_1\epsilon_f\overline{\nu\gamma}\epsilon_g\epsilon_n(\tau_2)\chi(\tau_2)\delta\left(\tau_3 - \frac{d_3}{c}\right)$$

$$C_{fnn}(\tau_2,\tau_3) = N_1\epsilon_f\overline{\nu(\nu-1)}\epsilon_n(\tau_2)\chi(\tau_2)\epsilon_n(\tau_3)\chi(\tau_3)$$

The bicorrelation $C_{123}(\tau_2,\tau_3)$ is essentially the product of $C_2(\tau_2)$ and $C_3(\tau_3|\tau_2))$. It is assumed that the only dependence between the correlations is the reduction in the number of the available radiations after the other detection. The reduction in the number of radiations is only true if the detectors are separated by a considerable distance. It is not improbable for a single radiation to register in two adjacent detectors.

The assumption that the time distribution of the two correlations are independent turns out to be reasonable correct. In other words, if a fast neutron is detected in one

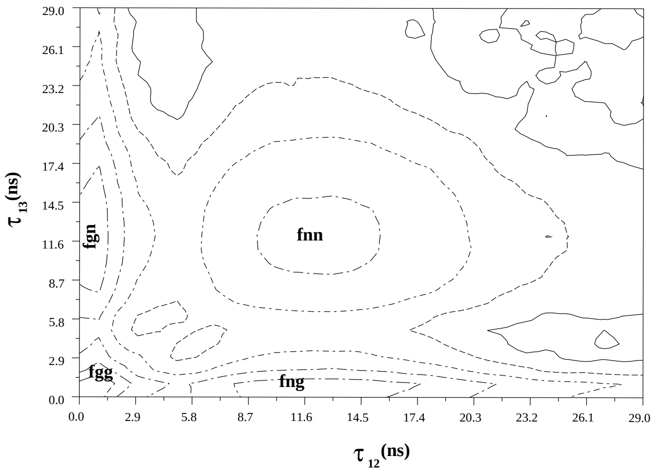

Figure 3: $C_{123}(\tau_2, \tau_3)$.

detector, the neutron detected in the second detector is no less likely to be fast.

Each of these terms can be seen in Figure 3. Each term is temporally separated from the others in the τ_2, τ_3 plane. Each of the four regions can be defined by rectangular bounds. The fissions correlated with gamma-gamma pairs form essentially a point $\tau_2 = \frac{d_2}{c}$ and $\tau_3 = \frac{d_3}{c}$ with a width of Δ_g. The fissions correlated with gamma-neutron pairs form lines parallel to the τ_2 and τ_3 axes and a width of Δ_g. The beginning and end of these lines are marked by fast n_f and threshold n_t neutrons respectively. The fission-neutron-neutron triplets are bounded by a rectangle formed from the delay times of each of the four pairs of fast and threshold neutrons.

A cross correlation C_{23} between two detectors is shown in Figure 4. A related detector-detector correlation $C_{23|1}$ can be derived from the source-two-detector correlation by a substitution of variables $\tau_{23} = \tau_3 - \tau_2$. This correlation is conditional on a fission being detected in in Channel 1. The boundaries of the four regions of this correlation are determined from this substitution of variables. This correlation can be thought of as the shadow of the bicorrelation C_{123} projected on the τ_3 axis.

$$\begin{aligned} C_{23|1}(\tau_{23}) &= \int_0^\infty C_{123}(\tau_2, \tau_{23} + \tau_2) d\tau_2 \\ &= C_{gg|f_1}(\tau_{23}) + C_{gn|f_1}(\tau_{23}) + C_{ng|f_1}(\tau_{23}) + C_{nn|f_1}(\tau_{23}) \end{aligned} \tag{13}$$

where

$$C_{gg|f_1}(\tau_{23}) = N_1\epsilon_f\overline{\gamma(\gamma-1)}\epsilon_g^2\int_0^\infty \delta\left(\tau_2 - \frac{d_2}{c}\right)\delta\left(\tau_{23}+\tau_2-\frac{d_3}{c}\right)d\tau_2$$

$$= N_1\epsilon_f\overline{\gamma(\gamma-1)}\epsilon_g^2\delta\left(\tau_{23} - \frac{d_3-d_2}{c}\right)$$

$$C_{gn|f_1}(\tau_{23}) = N_1\epsilon_f\overline{\nu\gamma}\epsilon_g\int_0^\infty \delta\left(\tau_2 - \frac{d_2}{c}\right)\epsilon_n(\tau_{23}+\tau_2)\chi(\tau_{23}+\tau_2)d\tau_2$$

$$= N_1\epsilon_f\overline{\nu\gamma}\epsilon_g\epsilon_n\left(\tau_{23} - \frac{d_2}{c}\right)\chi\left(\tau_{23} - \frac{d_2}{c}\right)$$

$$C_{ng|f_1}(\tau_{23}) = N_1\epsilon_f\overline{\nu\gamma}\epsilon_g\int_0^\infty \epsilon_n(\tau_2)\chi(\tau_2)\delta\left(\tau_{23}+\tau_2-\frac{d_3}{c}\right)d\tau_2$$

$$= N_1\epsilon_f\overline{\nu\gamma}\epsilon_g\epsilon_n\left(\frac{d_3}{c} - \tau_{23}\right)\chi\left(\frac{d_3}{c} - \tau_{23}\right)$$

$$C_{nn|f_1}(\tau_{23}) = N_1\epsilon_f\overline{\nu(\nu-1)}\int_0^\infty \epsilon_n(\tau_2)\chi(\tau_2)\epsilon_n(\tau_{23}+\tau_2)\chi(\tau_{23}+\tau_2)d\tau_2$$

The only difference in the case of the cross correlation C_{23} is that the fissions need not be observed. This can be designated by substituting N for N_1 and setting ϵ_f to one. This implies that $\frac{C_{23|1}}{C_{23}} = \frac{N_1\epsilon_f}{N}$. In addition, if the observed fission source N_1 is the only fission source then $N_1 = N$.

Finally, a correlation between three detectors can be measured. Such a correlation is shown in Figure 5. An expression for this correlation can be written as

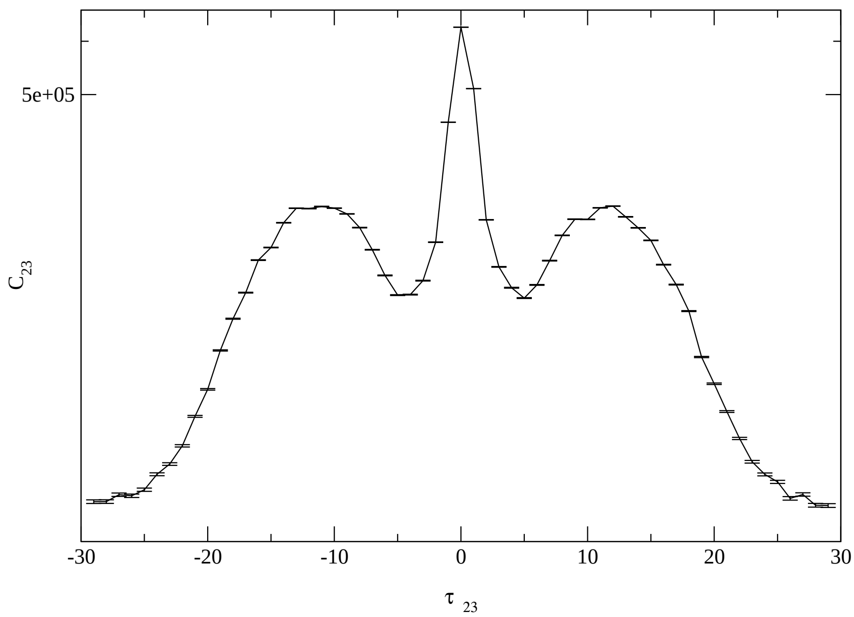

Figure 4: $C_{23}(\tau_{23})$.

$$
\begin{aligned}
C_{234}(\tau_{23},\tau_{24}) = {} & C_{ggg}(\tau_{23},\tau_{24}) \\
& + C_{ggn}(\tau_{23},\tau_{24}) + C_{gng}(\tau_{23},\tau_{24}) + C_{ngg}(\tau_{23},\tau_{24}) \\
& + C_{gnn}(\tau_{23},\tau_{24}) + C_{ngn}(\tau_{23},\tau_{24}) + C_{nng}(\tau_{23},\tau_{24}) \\
& + C_{nnn}(\tau_{23},\tau_{24})
\end{aligned}
\tag{14}
$$

where the eight terms are defines as

$$C_{ggg}(\tau_{23},\tau_{24}) = N\overline{\gamma(\gamma-1)(\gamma-2)}\epsilon_g^3 \int_0^\infty \delta\left(\tau_2 - \frac{d_2}{c}\right)\delta\left(\tau_{23}+\tau_2-\frac{d_3}{c}\right)\delta\left(\tau_{24}+\tau_2-\frac{d_4}{c}\right)d\tau_2$$

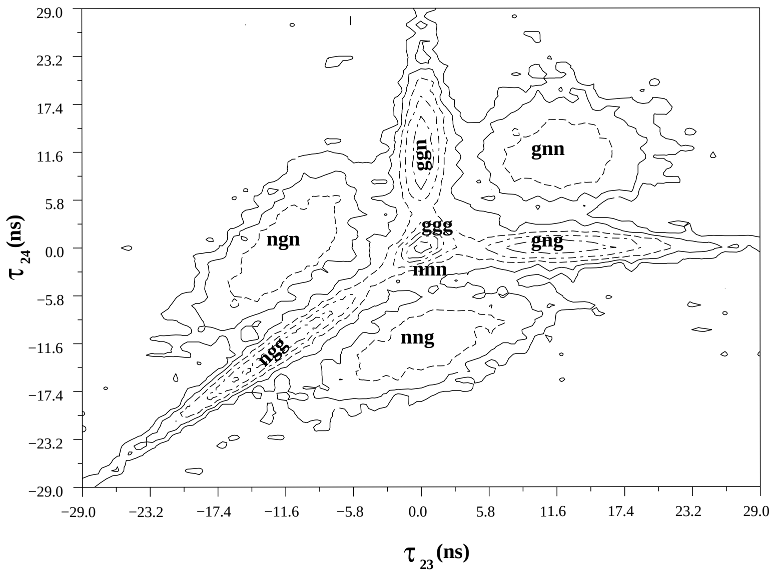

Figure 5: $C_{234}(\tau_{23}, \tau_{24})$.

$$= N\overline{\gamma(\gamma-1)(\gamma-2)}\epsilon_g^3)\delta\left(\tau_{23} - \frac{d_3 - d_2}{c}\right)\delta\left(\tau_{24} - \frac{d_4 - d_2}{c}\right)$$

$$C_{ggn}(\tau_{23}, \tau_{24}) = N\overline{\nu}\overline{\gamma(\gamma-1)}\epsilon_g^2 \int_0^\infty \delta\left(\tau_2 - \frac{d_2}{c}\right)\delta\left(\tau_{23} + \tau_2 - \frac{d_3}{c}\right)\epsilon_n(\tau_{24} + \tau_2)\chi(\tau_{24} + \tau_2)d\tau_2$$

$$= N\overline{\nu}\overline{\gamma(\gamma-1)}\epsilon_g^2)\delta\left(\tau_{23} - \frac{d_3 - d_2}{c}\right)\epsilon_n\left(\tau_{24} - \frac{d_2}{c}\right)\chi\left(\tau_{24} - \frac{d_2}{c}\right)$$

$$C_{gng}(\tau_{23}, \tau_{24}) = N\overline{\nu}\overline{\gamma(\gamma-1)}\epsilon_g^2 \int_0^\infty \delta\left(\tau_2 - \frac{d_2}{c}\right)\epsilon_n(\tau_{23} + \tau_2)\chi(\tau_{23} + \tau_2)\delta\left(\tau_{24} + \tau_2 - \frac{d_4}{c}\right)d\tau_2$$

$$= N\overline{\nu}\overline{\gamma(\gamma-1)}\epsilon_g^2)\epsilon_n\left(\tau_{23} - \frac{d_2}{c}\right)\chi\left(\tau_{23} - \frac{d_2}{c}\right)\delta\left(\tau_{24} - \frac{d_3 - d_2}{c}\right)$$

$$C_{ngg}(\tau_{23}, \tau_{24}) = N\overline{\nu}\overline{\gamma(\gamma-1)}\epsilon_g^2 \int_0^\infty \epsilon_n(\tau_2)\chi(\tau_2)\delta\left(\tau_{23} + \tau_2 - \frac{d_3}{c}\right)\delta\left(\tau_{24} + \tau_2 - \frac{d_4}{c}\right)d\tau_2$$

$$C_{gnn}(\tau_{23},\tau_{24}) = N\overline{\nu(\nu-1)}\overline{\gamma}\epsilon_g \int_0^\infty \delta\left(\tau_2 - \frac{d_2}{c}\right)\epsilon_n(\tau_{23}+\tau_2)\chi(\tau_{23}+\tau_2)\epsilon_n(\tau_{24}+\tau_2)\chi(\tau_{24}+\tau_2)d\tau_2$$

$$C_{ngn}(\tau_{23},\tau_{24}) = N\overline{\nu(\nu-1)}\overline{\gamma}\epsilon_g \int_0^\infty \epsilon_n(\tau_2)\chi(\tau_2)\delta\left(\tau_{23}+\tau_2 - \frac{d_3}{c}\right)\epsilon_n(\tau_{24}+\tau_2)\chi(\tau_{24}+\tau_2)d\tau_2$$

$$C_{nng}(\tau_{23},\tau_{24}) = N\overline{\nu(\nu-1)}\overline{\gamma}\epsilon_g \int_0^\infty \epsilon_n(\tau_2)\chi(\tau_2)\epsilon_n(\tau_{23}+\tau_2)\chi(\tau_{23}+\tau_2)\delta\left(\tau_{24}+\tau_2 - \frac{d_4}{c}\right)d\tau_2$$

$$C_{nnn}(\tau_{23},\tau_{24}) = N\overline{\nu(\nu-1)(\nu-2)} \int_0^\infty \epsilon_n(\tau_2)\chi(\tau_2)\epsilon_n(\tau_{23}+\tau_2)\chi(\tau_{23}+\tau_2)\epsilon_n(\tau_{24}+\tau_2)\chi(\tau_{24}+\tau_2)d\tau_2$$

The bicorrelation $C_{234}(\tau_{23},\tau_{24})$ has eight regions formed by all combinations of gamma and neutron detections in the three detectors. It is the product of the three correlations $C_2(\tau_2)$, $C_3(\tau_3)$ and $C_4(\tau_4)$ and a substitution of the variables $\tau_{23} = \tau_3 - \tau_2$ and $\tau_{24} = \tau_4 - \tau_2$. The triplets of gammas essentially form a point with a width of Δ_g. The boundary forms a hexagon from six of the eight combinations of $\pm\frac{\Delta_g}{2}$. The all minus and all plus combinations occur within the interior of this region. The triplets of neutrons are also bounded by a hexagon formed by the six combinations of fast and threshold neutrons. The two combinations with three fast and three threshold neutrons also occur within the boundary.

There are three regions comprised of two gammas correlated with one neutron. Each of these regions form essentially a line with a width of Δ_g.

5.2 Multiplicity data

As in NMC, the neutron multiplicities are treated as physical constants which can be found in the literature.[21]

The gamma ray multiplicity distribution from fission, $\gamma(G)$ can be computed using varying models.[23] One model is the double Poisson model used by Brunson[7]. The major drawback with this model is that three constants, must be determined. A model that only requires two parameters is the negative binomial distribution[25].

5.3 Ratios of detector-detector correlations in passive measurement

In various ratios of the form $\frac{C_{ij}(\tau_{ij})}{C_{ij|k}(\tau_{ij})}$ are possible. For three detectors, 2, 3, 4, there are three conditional correlations contained in the bicorrelation $C_{234}(\tau_{23},\tau_{24})$. Using the relation $\tau_{34} = \tau_{23} - \tau_{24}$, These three conditional correlations are

$$
\begin{aligned}
C_{23|4}(\tau_{23}) &= \int_0^\infty C_{234}(\tau_{23},\tau_{24})d\tau_{24} \\
C_{24|3}(\tau_{24}) &= \int_0^\infty C_{234}(\tau_{23},\tau_{24})d\tau_{23} \\
C_{34|2}(\tau_{34}) &= \int_0^\infty C_{234}(\tau_{34}+\tau_{24},\tau_{24})d\tau_{24} \\
&= \int_0^\infty C_{234}(\tau_{23},\tau_{23}-\tau_{34})d\tau_{23}
\end{aligned}
$$

In addition, the correlations can be separated by reaction type. For example $C_{ij|k}(gg|n)$ represents a gamma detection in both detector i and j given that a neutron was detected in detector k. This conditional correlation is a function of τ_{ij} and comes from doing the appropriate integration on C_{ijk}. It can also conveniently be written as $C_{gg|n}(\tau_{ij})$.

$$\frac{C_{gg|n}}{C_{gg}} = \overline{\nu}\epsilon_n \tag{15}$$

$$\frac{C_{gn|g}(\tau_{ij})}{C_{gn}(\tau_{ij})} = \frac{\overline{\gamma(\gamma-1)}}{\overline{\gamma}}\epsilon_g \tag{16}$$

$$\frac{C_{gn|n}(\tau_{ij})}{C_{gn}(\tau_{ij})} = \frac{\overline{\nu(\nu-1)}}{\overline{\nu}}\epsilon_n(\tau_{ij}) \tag{17}$$

$$\frac{(C_{gn}(\tau_{ij}))^2}{C_{gn|g}(\tau_{ij})} = N\frac{\overline{\gamma}^2\overline{\nu}}{\overline{\gamma(\gamma-1)}}\epsilon_n(\tau_{ij}) \tag{18}$$

$$\frac{(C_{gn}(\tau_{ij}))^2}{C_{gn|n}(\tau_{ij})} = N\frac{\overline{\nu}^2\overline{\gamma}}{\overline{\nu(\nu-1)}}\epsilon_g \tag{19}$$

$$\frac{C_{gg}C_{gn}}{C_{gn|g}} = N\overline{\gamma}\epsilon_g \tag{20}$$

$$\frac{C_{nn}(\tau_{ij})C_{gn}(\tau_{ij})}{C_{gn|n}(\tau_{ij})} = N\overline{\nu}\epsilon_n(\tau_{ij}) \tag{21}$$

$$\frac{C_{gg}}{C_{ggg}} = \frac{\overline{\gamma(\gamma-1)}}{\overline{\gamma(\gamma-1)(\gamma-2)}}\epsilon_g \tag{22}$$

These ratios can be used to determine the fissile mass. The number of fissions counted are explicit in Equations (18)(19)(20) and (21). Detection efficiency for gamma rays and neutrons are isolated in a number of the equations.

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